Bruno Mera (Universidade de Lisboa and AIMR, Tohoku University)

Title: From ideal bands to generalized Landau levels: quantum geometry of Bloch bands in the holomorphic setting

Abstract: This talk aims to provide an introduction to the notion of Generalized Landau Levels (GLLs) from a geometric point of view. GLLs are Bloch bands that generalize the standard notion of Landau levels of a charged particle in a uniform magnetic field. Beginning with a foundational introduction to quantum geometry—the differential geometry of families of quantum states—I will delve into the specific case of Bloch bands, unraveling the inequalities that emerge relating the quantum metric and the Berry curvature, the saturation of which implies holomorphicity and gives rise to the concept of *Kähler band*. A Kähler band can then be understood as a regular holomorphic curve in complex projective space. The geometry of holomorphic curves shares many properties with that of real curves in Euclidean space. In particular, there is a distinguished moving frame along the curve, *the Frenet-Serret frame* (unique up to a global phase), whose elements are the GLLs. The frame satisfies the so-called Frenet-Serret equations which, together with the Maurer-Cartan structure equation, allow us not only to derive the quantum geometry of each GLL but also to establish geometric recursion relations among them. The content of these recursion relations is a manifestation of Calabi's rigidity theorem for Kähler immersions into projective space that, in this language, not only establishes the uniqueness, up to a momentum-independent unitary transformation, of a Kähler band with a given Berry curvature profile, but also completely determines the quantum geometry of the GLLs. As a natural consequence, the quantum volume of the quantum metric of the *n*th GLL is exactly quantized to *2n+1*. The discussion finds direct applications to moiré materials, where the 0th GLL, the Kähler band, and the 1st GLL are bands which can stabilize fractional Abelian and non-Abelian, respectively, fractional Chern insulating phases.