

Topics

1. The fractional Laplacian. Probabilistic meaning and basic properties.
2. The extension problem. Weak solutions and energy. The Hamiltonian. The Dirichlet problem.
3. The fractional Allen-Cahn equation and nonlocal minimal surfaces.
4. Curves and surfaces with constant nonlocal mean curvature.

Abstract

In these talks I will explain basic ideas and recent developments concerning fractional Laplacians and other nonlocal integro-differential equations. We will start with their basic properties (their relation with probability, with fractional Sobolev spaces, and the extension problem associated to them). Later we will treat some important nonlinear equations involving these operators: the fractional Allen-Cahn equation, the notion of fractional perimeter, and the concept of nonlocal mean curvature.

The regularity theory and the nonlinear analysis for these equations is one of the most active fields in PDEs nowadays. The main advances have been made in the last decade. Still many remain to be discovered. The interest and applications of these equations is extremely large, not only within mathematics but mainly in other fields (probability and mathematical finance, image processing, biological invasions, anomalous fluids, geophysical sciences, etc.)