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Monodromies of hyperelliptic families of genus three curves

by

Mizuho Ishizaka

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A thesis presented

by

Mizuho Ishizaka

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Monodromies of hyperelliptic families of genus three curves

Mizuho ISHIZAKA

Contents

Introduction

Let $\phi\colon S \longrightarrow \Delta$ be a proper surjective holomorphic map from a complex surface S to a small disk $\Delta = \{t \in \mathbf{C} |\ |t| < \epsilon\}$ such that $\phi^{-1}(t)$ is a smooth curve of genus $g \geq 2$ for any $t \in \Delta^* = \Delta \setminus \{0\}$. We call (ϕ, S, Δ) a *degeneration of curves.* If all $\phi^{-1}(t)$ for $t \in \Delta^*$ are hyperelliptic curves, we call (ϕ, S, Δ) a *hyperelliptic family.* If the reduced scheme of the special fiber has normal crossing and any (-1)-curve in the special fiber intersects the other components at at least three points, we call (ϕ, S, Δ) *normally minimal.* Two degenerations (ϕ, S, Δ) and (ϕ', S', Δ) are said to be *topologically equivalent* if there exists an orientation-preserving homeomorphism $\psi\colon S \longrightarrow S'$ which satisfies $\phi' \circ \psi = \phi$. Let $\mathcal{T}_g := \{$ normally minimal degenerations of genus g $\}/\sim$, where $\sim$ is the topological equivalent relation. For an element of $\mathcal{T}_g$, we can uniquely determine the topological monodromy (sometimes called monodromy, for short) as a conjugacy class in the mapping class group of genus g. The monodromy of a degeneration is a conjugacy class of a pseudo-periodic map of negative type (cf. [MM1], [Ni1], [Ni2], [Im], [ES], [ST], [AMO] etc.). Conversely, any conjugacy class of a pseudo-periodic map of negative type is realized as the monodromy of a certain degeneration (cf. [MM2]). In [AI], we classified the monodromies of degenerations of curves of genus three with their topological types of moduli points. In this paper, we classify the monodromies of degenerations of genus three which are realized as the monodromies of certain hyperelliptic families. In Section 1, we review the results of Matsumoto-Montesinos theory (cf. [MM1], [MM2]). In Section 2, we review the results of classification of degenerations of curves of genus three via Matsumoto-Montesinos theory (cf. [AI]) and introduce several symbols which are used in Section 3 and Section 4. In Section 3 we classify the monodromies which *cannot be realized* as the monodromy of any hyperelliptic family of genus three (Theorems 3.2, 3.3, 3.6). Let $\overline{\mathbf{M}}_3$ be the Deligne-Mumford compactification of the moduli space of curves of genus three. Theorem 3.2 also states that the closure of hyperelliptic locus in $\overline{\mathbf{M}}_3$ has no intersection with the part of boundaries of the loci of the stable curves whose topological types are (D), (H), (I), (M) and (O) in Table 2. In Section 4, for each monodromy which can be realized as the topological monodromy of a hyperelliptic family of genus three, we give an example of the equation of the hyperelliptic family.

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1 Matsumoto-Montesinos theory

In this section, we review Matsumoto-Montesinos theory for later use.

Let $f\colon \Sigma_g \longrightarrow \Sigma_g$ be an orientation-preserving homeomorphism of a closed surface (real two dimensional manifold) of genus g. We call f a pseudo-periodic map if f is isotopic to a homeomorphism $f'\colon \Sigma_g \longrightarrow \Sigma_g$ such that the following conditions are satisfied:

(i) There exists a disjoint union of simple closed curves $\mathcal{C} = C_1 \cup C_2 \cup \ldots \cup C_r$ on the interior of Σ such that $f'(\mathcal{C}) = \mathcal{C}$ ($\mathcal{C}$ might be empty).

(ii) Set $\mathcal{B} = \Sigma_g - \mathcal{C}$. Then the restriction $f'|_{\mathcal{B}}\colon \mathcal{B} \to \mathcal{B}$ is isotopic to a periodic map, i.e., there is a positive integer N such that $(f'|_{\mathcal{B}})^N$ is isotopic to the identity map.

Suppose f is a pseudo-periodic map by a system of simple closed curves $\mathcal{C} = C_1 \cup C_2 \cup \ldots \cup C_r$. $\mathcal{C}$ is called *admissible* if each connected component of $\Sigma_g \setminus \mathcal{C}$ has negative Euler number. If $g \geq 2$, such a system always exists. We sometimes write $\overrightarrow{C}_i$ to emphasize its orientation. For each component C_i of $\mathcal{C}$, there exists a minimal integer α_i such that $f^{\alpha_i}(\overrightarrow{C}_i) = \overrightarrow{C}_i$. There also exists a minimal positive integer L_i such that $f^{L_i}|_{C_i}$ is a Dehn twist of e_i times ($e_i \in \mathbf{Z}$). We set $s(C_i) = e_i\alpha_i/L_i$ and call it the *screw number* of f at C_i (cf. [Ni2]). An admissible system $\mathcal{C}$ is *precise* if $s(C_i) \neq 0$ for each C_i. A precise system always exists and is unique up to isotopy. We say that a pseudo-periodic map f is *of negative twist* if $s(C_i) < 0$ for each curve C_i in a precise system $\mathcal{C}$. A curve C_i is said to be *amphidrome* if α_i is even and $f^{\alpha_i/2}(\overrightarrow{C}_i) = -\overrightarrow{C}_i$.

For a degeneration (ϕ, S, Δ) of genus g, by fixing a base point $t_0 \in \Delta \setminus \{0\}$, the canonical generator of $\pi(\Delta \setminus \{0\}, t_0) \simeq \mathbf{Z}$ acts naturally on the Riemann surface $\phi^{-1}(t_0)$, i.e., there exists an orientation preserving homeomorphism $F_{\phi,t_0}\colon \phi^{-1}(t_0) \longrightarrow \phi^{-1}(t_0)$ and F_{ϕ,t_0} is uniquely determined up to isotopy. We see that F_{ϕ,t_0} is a pseudo-periodic map of negative twist by the result of Imayoshi [Im], Shiga and Tanigawa [ST], and Earle and Sipe [ES]. Since a change of the base point corresponds to a conjugation in the mapping class group $\mathcal{M}_g$, a degeneration (ϕ, S, Δ) uniquely determines an element $m(\phi)$ of the set of conjugacy classes $\widehat{\mathcal{M}}_g$ of $\mathcal{M}_{\mathbf{g}}$. We call $m(\phi)$ the *topological monodromy* of (ϕ, S, Δ). Let $\mathcal{P}_g^-$ denote the subset of $\widehat{\mathcal{M}}_g$ represented by pseudo-periodic map of negative twist. Since $m(\phi)$ is determined by the class of (ϕ, S, Δ) in $\mathcal{T}_g$, we have well-defined map

$$\gamma\colon \mathcal{T}_g \longrightarrow \mathcal{P}_g^-.$$

Theorem 1.1 ([MM2, Theorem 1]) *If $g > 1$, then $\gamma\colon \mathcal{T}_g \longrightarrow \mathcal{P}_g^-$ is bijective.*

In the case of $g = 1$, γ is surjective but not injective. Let (ϕ_i, S_i, Δ) $(i = 1, 2)$ be degeneration of curves of genus one and let F_i be the special fibers of (ϕ_i, S_i, δ). If there exists a positive integer m such that F_1 is equal to mF_2 as the numerical chorizo space (the topological space considering the multiplicities of their components), $\gamma(\phi_1, S_1, \Delta) = \gamma(\phi_2, S_2, \Delta)$.

Corollary 1.2 [MM2, Corollary 1.2] *Given a pseudo-periodic map $f\colon \Sigma_g \longrightarrow \Sigma_g$ of negative twist, there exists a degeneration (ϕ, S, Δ) whose monodromy is the conjugacy class of f in the mapping class group $\mathcal{M}_g$.*

The outline of the proof of Theorem 1.1 is as follows:

Step 1 For a pseudo-periodic map of negative twist f, they construct a numerical chorizo space $\mathcal{S}_f$ called *generalized quotient*. Note that there is a natural projection $\pi\colon \Sigma_g \longrightarrow \mathcal{S}_f$.

Step 2 Let $C_\pi = \{\Sigma_g \times [0,1] \cup \mathcal{S}_f\}/(x,0) \sim \pi(x)$ be the mapping cylinder of π. The super standard form F of $f\colon \Sigma_g \longrightarrow \Sigma_g$ can be extended to an automorphism of C_π (cf. [MM1]). They construct an open book $\overline{M} = \{[0, 2\pi] \times C_\pi\}/\sim$, where the equivalent relation "$\sim$" is defined as follows (cf. [Ta], [W]):

$$(2\pi, c) \sim (0, F(c)) \ (c \in C_\pi), \quad (\theta, c) \sim (0, c) \ (c \in S_f, \theta \in [0, 2\pi])$$

Note that there is the canonical map $\phi\colon \overline{M} \longrightarrow \overline{\Delta}$, where $\overline{\Delta} = \{t \in \mathbf{C} | |t| \leq 1\}$.

Step 3 $M := \phi^{-1}(\Delta)$ has a complex structure, and we obtain a degeneration (ϕ, M, Δ) whose monodromy is the conjugacy class of f in $\mathcal{M}_g$.

Note that the special fiber of (ϕ, M, Δ) is $\mathcal{S}_f$. Since the configuration of the special fiber is very important to classify the monodromies which are realized as the monodromies of certain hyperelliptic families, we introduce the construction of the generalized quotient S_f of a pseudo-periodic map f of negative twist.

Let $\mathcal{A}$ be the union of annular neighborhoods of the curves in the precise system $\mathcal{C}$ such that $f(\mathcal{A}) = \mathcal{A}$ and let $\mathcal{B}$ be the closure of $\Sigma \setminus \mathcal{A}$. We may assume that $f|_{\mathcal{B}}\colon \mathcal{B} \longrightarrow \mathcal{B}$ is periodic, i.e., there exists a positive integer N such that $f|_{\mathcal{B}}^N$ is isotopic to the identity of $\mathcal{B}$. The quotients space $\mathcal{B}/(f|_{\mathcal{B}})$ is an orbifold. We call $p \in \mathcal{B}/(f|_{\mathcal{B}})$ a *cone point* if the number of points of $f|_{\mathcal{B}}^{-1}(p)$ is not N. Let $p \in \mathcal{B}/(f|_{\mathcal{B}})$ and (m, λ, σ) be a cone point and the *valency* of p, respectively (cf. [Ni1]). The valency (m, λ, σ) of p has the following topological meaning: If $x \in f|_{\mathcal{B}}^{-1}(p)$ and m is the smallest positive integer such that $f^m(x) = x$, f^m is the rotation around x through the angle $2\pi\delta/\lambda$ $(0 < \delta < \lambda,\ \gcd(\lambda, \delta) = 1)$ and σ is the integer determined by $\delta\sigma \equiv 1 \pmod{\lambda}$, $0 < \sigma < \lambda$. By the Euclidean algorithm we obtain a sequence of integers $n_0 > n_1 > \cdots > n_l = 1$ such that $n_0 = \lambda$, $n_1 = \lambda - \sigma$, $n_{i-1} + n_{i+1} \equiv 0 \pmod{n_i}$, $i = 1, 2, \cdots, l-1$. Set $m_i = mn_i$ $(i = 0, 1, \cdots, l)$. Let $\mathrm{Ch}(\mathcal{B})$ be the chorizo space constructed from $\mathcal{B}/(f|_{\mathcal{B}})$ by replacing a neighborhood of each cone point with the numerical chorizo space shown in Figure A, which consists of a disk and l spheres. Let $\mathcal{A}_i (\subset \mathcal{A})$ be an annular neighborhood of C_i. The boundary curves S_1 and S_2 of A_i have their valencies $(m^{(1)}, \lambda^{(1)}, \sigma^{(1)})$ and $(m^{(2)}, \lambda^{(2)}, \sigma^{(2)})$, respectively, when we regard them as boundary curves of the periodic part $\mathcal{B}$ (cf. [Ni1]).

Suppose C_i is not amphidrome. Then $m^{(1)} = m^{(2)} = \alpha(C_i)$. Let m be this common value.

Lemma 1.3 ([MM1 Theorem 6.1]) *There exists uniquely a sequence of positive integers $n_0, n_1, \cdots, n_l$ $(l \geq 1)$ satisfying the following conditions:*

(i) $n_0 = \lambda^{(1)}$ *and* $n_l = \lambda^{(2)}$,

(ii) $n_1 \equiv \sigma^{(1)} \pmod{\lambda^{(1)}}$ *and* $n_2 \equiv \sigma^{(2)} \pmod{\lambda^{(2)}}$,

(iii) $n_{i-1} + n_{i+1} \equiv 0 \pmod{n_i}$ $(i = 1, 2, \cdots, l-1)$,

(iv) $(n_{i-1} + n_{i+1})/n_i \geq 2$ $(i = 1, 2, \cdots, l-1)$,

(v) $\sum_{i=1}^{l-1} 1/n_i n_{i+1} = |s(C_j)|$.

Let $\mathrm{Ch}(\mathcal{A}_i)$ be the chorizo space shown in Figure B, which consists of two disks and $l-1$ spheres. The multiplicity m_i of each components is defined to be mn_i. We consider that the spaces are $\mathrm{Ch}(f^j\mathcal{A}_i)$ $(j = 0, 1, \cdots, m-1)$ are identical, i.e., $\mathrm{Ch}(\mathcal{A}_i) = \mathrm{Ch}(f\mathcal{A}_i) = \cdots = \mathrm{Ch}(f^{m-1}\mathcal{A}_i)$. Suppose C_i is amphidrome. Then S_1 and S_2 have the same valency $(2m, \lambda, \sigma)$, where $2m = \alpha(C_j)$. We can determine a sequence of positive integers $n_0 \geq n_1 \geq \cdots \geq n_l = 1$ in similar way in the case of non-amphidrome and let $\mathrm{Ch}(\mathcal{A}_i)$ be the chorizo space shown in Figure C, which consists of a disk and $l+2$ spheres. We also consider that the spaces $\mathrm{Ch}(f^j\mathcal{A}_i)$ $(i = 0, 1, \cdots, (m/2)-1)$ are identical. Now the generalized quotient S_f is defined to be the union of $\mathrm{Ch}(\mathcal{B})$ and $\mathrm{Ch}(\mathcal{A}_i)$'s.

2 Classification of monodromies of genus three

In this section, we introduce the result of [AI] and define the symbols which we use in Section 3 and Section 4.

2.1 Algorithm for classification

We classified degenerations of curves of genus three in [AI]. Our basic tools are Corollary 1.2 and the following theorem:

Theorem 2.1 [MM2, Theorem 2] *The conjugacy class of a pseudo-periodic map of negative type* $f\colon \Sigma_g \longrightarrow \Sigma_g$ *of genus* g *is determined by the following data: A precise admissible system of cut curves* $\mathcal{C} = \coprod C_i$ *on* Σ_g*, the action of* f *on the weighted oriented graph* $G_{\mathcal{C}}$ *induced by* $\mathcal{C}$*, the screw numbers of* f *at each* C_i *and the valency data of the periodic maps which stabilize the connected components of* $\Sigma_g - \mathcal{C}$*.*

The classification is divided into the following four steps:

Step 1 We classify admissible systems $\mathcal{C}$ of cut curves on Σ_3 (cf. [AI, Lemma 3.2], Table 2 of this paper). This is equivalent to classifying the stable curves of genus three, which is well-known. We disregard the orientations of edges in Table 2 in this step.

Step 2 We classify cyclic automorphisms of the weighted oriented graphs $G_{\mathcal{C}}$ in Table 2 (cf. [AI, Lemma 3.4]). This is equivalent to classifying cyclic automorphisms of the dual graphs of stable curves of genus three. In order to study the amphidrome action of a pseudo-periodic map, we must care about the orientations of edges in Table 2.

Step 3 We classify periodic homeomorphisms modulo isotopy of a Riemann surface with boundary, which is realized as a connected component of $\Sigma_3 - \mathcal{C}$. This is equivalent to classifying cyclic automorphisms of an irreducible n-pointed stable curve of genus at most three which admit permuting marked points. For this purpose, we first classify the valency data of the cyclic automorphisms of a closed Riemann surface of genus at most three by using Nielsen's theorem, and Harvey's formula (cf. [AI, Lemma 1.4]). The sufficiency of these conditions is proved in [AI, §4.4].

Step 4 For each cyclic automorphism of the graph $G_{\mathcal{C}}$ classified in Step 2, we determine the compatible periodic homeomorphisms of "parts" of a Riemann surface described in Step 3. In this way, we classify all conjugacy classes of pseudo-periodic maps of negative type of genus 3 (cf. [AI, Proposition 3.8]).

Table 2 is introduced for Step 1. The weighted graphs (A) through (O) in Table 2 are the dual graphs of stable curves of genus three (cf. [AI, Lemma 3.2]). A vertex v corresponds to a component of a stable curve and an edge connecting two vertices corresponds to an intersection point of the two components. Let $g(v)$ be the genus of v and let $\rho(v)$ be the number of singular points of the component. The number inside a small circle of the graph is the arithmetic genus $g(v)+\rho(v)$ of v. We omit the number when it is zero. For instance, the graph (B) represents six kinds of stable curves, that is, v_1 has genus i_1 and $2-i_1$ singular points while v_2 has genus i_2 and $1-i_2$ singular points $(0 \leq i_1 \leq 2,\ 0 \leq i_2 \leq 1)$. We write these stable curves $B_{i_1 i_2}$ $(0 \leq i_1 \leq 2,\ 0 \leq i_2 \leq 1)$ for short. Remark that considering that an edge corresponds to a component of an admissible system, we see that Table 2 gives the classification of the decomposition of Σ_3 by a precise admissible system.

For Step 2, we must define some symbols. For an example, consider the graph of type E_{11}. We have two non-trivial automorphisms σ as follows:

(1) σ fixes the vertices v_1 and v_2, and interchanges the edges e_1 and e_2 preserving their orientations.
(2) σ interchanges v_1 and v_2 , and fixes e_1 and e_2 as a set, but changes their orientations.
(3) σ interchanges the vertices v_1 and v_2 and interchanges the edges e_1 and e_2.

We express by $\langle (e_1, e_2) \rangle$ the cyclic group generated by the automorphism (1), express by $\langle (v_1, v_2)(e_1, -e_1)(e_2, -e_2) \rangle$ the cyclic group generated by the automorphism (2), and express by $\langle (v_1, v_2)(e_1, -e_2)(e_2, -e_1) \rangle$ the cyclic group generated by the automorphism (3). This is equivalent to the following: We consider the symmetric group S_6 of formally independent six elements $v_1,\ v_2,\ e_1,\ -e_1,\ e_2,\ -e_2$. Then the group $\langle (e_1, e_2) \rangle$ is the subgroup of S_6 generated by $(e_1, e_2)(-e_1, -e_2)$. The group $\langle (v_1, v_2)(e_1, -e_1)(e_2, -e_2) \rangle$ is the subgroup generated by $(v_1, v_2)(e_1, -e_1)(e_2, -e_2)$.

In order to express all cyclic automorphism groups of each weighted oriented graphs in Table 2, we introduce the subgroups II(0,1) through V(1,1) of the symmetric group S_{17} of seventeen variables $v_1, \ldots, v_5, \pm e_1, \ldots, \pm e_6$;
II(0,1)= $\langle (e_1, e_2) \rangle$,
II(0,2)= $\langle (e_1, e_2)(e_3, e_4) \rangle$,
II(1,1)= $\langle (v_1, v_2)(e_1, e_2) \rangle$,
II(1,2)= $\langle (v_1, v_2)(e_1, -e_2) \rangle$,
II(1,3)= $\langle (v_1, v_2)(e_1, -e_1)(e_2, -e_2) \rangle$,
II(1,4)= $\langle (v_1, v_2)(e_1, -e_2)(e_3, e_4) \rangle$,
II(1,5)= $\langle (v_1, v_2)(e_1, -e_2)(e_3, -e_4) \rangle$,
II(1,6)= $\langle (v_1, v_2)(e_1, -e_1)(e_2, -e_2)(e_3, e_4) \rangle$,
II(1,7)= $\langle (v_1, v_2)(e_1, -e_1)(e_2, -e_2)(e_3, -e_4) \rangle$,

II(1,8)= $\langle (v_1, v_2)(e_1, -e_1)(e_2, -e_2)(e_3, -e_3)(e_4, -e_4)\rangle$,
II(1,9)= $\langle (v_1, v_2)(e_1, e_3)(e_2, e_4)(e_5, -e_5)\rangle$,
II(1,10)= $\langle (v_1, v_2)(e_1, e_2)(e_4, -e_4)(e_5, e_6)\rangle$,
II(2,1)= $\langle (v_1, v_2)(v_3, v_4)(e_1, -e_2)(e_3, e_4)\rangle$,
II(2,2)= $\langle (v_1, v_2)(v_3, v_4)(e_1, -e_1)(e_2, -e_2)(e_3, e_4)\rangle$,
II(2,3)= $\langle (v_1, v_2)(v_3, v_4)(e_1, -e_2)(e_3, -e_4)(e_5, e_6)\rangle$,
II(2,4)= $\langle (v_1, v_2)(v_3, v_4)(e_1, -e_1), (e_2, -e_2)(e_3, -e_3)(e_4, -e_4)(e_5, e_6)\rangle$,
II(2,5)= $\langle (v_1, v_2), (v_3, v_4), (e_1, -e_2), (e_3, -e_3)(e_4, -e_4)(e_5, e_6)\rangle$,
II(2,6)= $\langle (v_1, v_2)(v_3, v_4)(e_1, -e_5), (e_2, -e_6)(e_3, -e_3)(e_4, -e_4)\rangle$,
II(2,7)= $\langle (v_1, v_4)(v_2, v_3)(e_5, -e_6), (e_1, -e_3)(e_2, -e_4)\rangle$,
II(2,8)= $\langle (v_1, v_3)(v_2, v_4)(e_1, e_3), (e_2, e_4)(e_5, -e_5)(e_6, -e_6)\rangle$,
III(0,1)= $\langle (e_1, e_2, e_3)\rangle$,
III(1,1)= $\langle (v_1, v_2, v_3)(e_1, e_2, e_3)\rangle$,
III(1,2)= $\langle (v_1, v_2, v_3)(e_1, e_2, e_3)(e_4, e_5, e_6)\rangle$,
IV(0,1)= $\langle (e_1, e_2, e_3, e_4)\rangle$,
IV(1,1)= $\langle (v_1, v_2)(e_1, -e_2, e_3, -e_4)\rangle$,
IV(1,2)= $\langle (v_1, v_2)(e_1, e_2, e_3, e_4)(e_5, e_5)\rangle$,
IV(1,3)= $\langle (v_1, v_3, v_4, v_2)(e_6, e_3, -e_2, e_4)(e_1, e_5, -e_1, -e_5)\rangle$,
IV(2,1)= $\langle (v_1, v_4)(v_2, v_3)(e_1, -e_3, e_2, -e_4)(e_5, -e_6)\rangle$,
IV(2,2)= $\langle (v_1, v_3)(v_2, v_4)(e_1, e_3, e_2, e_4)(e_5, -e_5)(e_6, -e_6)\rangle$,
VI(1,1)= $\langle (v_1, v_2)(e_1, -e_2)(e_3, -e_1)(e_2, -e_3)(e_4, -e_4)\rangle$,

Consider the type E_{00} and E_{11}. Then the non-trivial cyclic automorphism groups of them are $\mathrm{II}(1,2)$ or $\mathrm{II}(1,3)$. For simplicity, we express it as

$$E_{ij}\ (0 \le i, j \le 1):\ \mathrm{II}(0,1),\ \mathrm{II}(1,2),\ \mathrm{II}(1,3).$$

Then, we have the following lemma.

Lemma 2.2 ([AI, Lemma 3.4])

The non-trivial cyclic automorphism groups of the weighted oriented graphs in Table 2 are classified as follows:

(1) C_{iij} $(0 \le i, j \le 1)$: II(1,1).
(2) D_{iij} $(0 \le i, j \le 1)$: II(1,1), D_{iii} $(0 \le i \le 1)$: III(1,1).
(3) E_{ij} $(0 \le i, j \le 1)$: II(0,1), E_{ii} $(0 \le i \le 1)$: II(0,1), II(1,2), II(1,3).
(4) F_{ij} $(0 \le i, j \le 1)$: II(0,1), F_{ii} $(0 \le i \le 1)$: II(0,1), II(2,1), II(2,2).
(5) G_{ij} $(0 \le i \le 1)$: II(0,1).

(6) H_i $(0 \le i \le 1)$: II(0,1), III(0,1).
(7) I_i $(0 \le i \le 1)$: II(0,1), III(0,1).
(8) J $(0 \le i \le 1)$: II(0,1), II(1,4), II(1,6).
(9) K $(0 \le i \le 1)$*:* II(0,1), II(1,4), II(1,6).
(10) L: II(0,1), II(0,2), II(1,5), II(1,7), II(1,8), III(0,1), IV(0,1), IV*(1,1),* VI*(1,1).*
(11) M: II(0,1), II(0,2), II(1,9), IV(1,2).
(12) N: II(0,1), II(0,2), II(2,3), II(2,4), II(2,5), II(2,7), II(2,8), IV(2,1), IV(2,2).
(13) O: II(1,10), II(2,6), III(1,2), IV(1,3)

2.2 Classification of periodic map

For Step 3, we classified the periodic automorphism of curves with r nodes and k boundaries (cf. [AA §2]).

First, we classify the periodic map of curves of genus at most three. In the isotopy class of a periodic map of Σ_g, one can choose a representative which is an analytic automorphism under a certain complex structure on Σ_g ([Ni1], [B2, Theorem 1] or in more generalized form [Ke]). Thus, it suffices to classify the cyclic analytic automorphism of curves of genus at most three. Let $f\colon \Sigma_g \to \Sigma_g$ be a cyclic analytic automorphism of order n, and let $\Pi\colon \Sigma \to \Sigma'$ be the corresponding n-fold cyclic covering. Let g' be the genus of Σ'.

By Nielsen's theorem [Ni1, §11], it suffices to classify the order of the map and the valencies of cone points and the valencies of boundaries. For brevity's sake, if we have the data of valencies $(n/\lambda_i, \lambda_i, \sigma_i)$ $(1 \le i \le l)$, we symbolically write $\sigma_1/\lambda_1 + \ldots + \sigma_l/\lambda_l$ which we call the *total valency.* We also write the order n of the map and the genus g' of Σ'. However, if $g' = 0$, the genus is omitted.

First, we classify the periods and total valencies of curves of genus at most two.

Lemma 2.3 [AI, Lemma 1.4]

Non-identical conjugacy classes of periodic maps of closed surfaces of genus g $(1 \le g < 3)$ *are classified followings:*
(I) $g = 1$*:*
(1) $n = 6$; $1/6 + 1/3 + 1/2$, $5/6 + 2/3 + 1/2$.
(2) $n = 4$; $1/4 + 1/4 + 1/2$, $3/4 + 3/4 + 1/2$.
(3) $n = 3$; $1/3 + 1/3 + 1/3$, $2/3 + 2/3 + 2/3$.
(4) $n = 2$; $1/2 + 1/2 + 1/2 + 1/2$.
(5) $g' = 1$, n is arbitrary and $\Pi\colon \Sigma \to \Sigma'$ is an unramified covering.

(II) $g = 2$:

(1) $n = 10$; $1/10 + 2/5 + 1/2$, $3/10 + 1/5 + 1/2$, $7/10 + 4/5 + 1/2$, $9/10 + 3/5 + 1/2$.

(2) $n = 8$; $1/8 + 3/8 + 1/2$, $5/8 + 7/8 + 1/2$.

(3) $n = 6$; $1/6 + 1/6 + 2/3$, $5/6 + 5/6 + 1/3$, $1/3 + 2/3 + 1/2 + 1/2$.

(4) $n = 5$; $1/5 + 1/5 + 3/5$, $1/5 + 2/5 + 2/5$, $2/5 + 4/5 + 4/5$, $3/5 + 3/5 + 4/5$.

(5) $n = 4$; $1/4 + 3/4 + 1/2 + 1/2$.

(6) $n = 3$; $1/3 + 1/3 + 2/3 + 2/3$.

(7) $n = 2$; $1/2 + 1/2 + 1/2 + 1/2 + 1/2 + 1/2$.

(8) $g' = 1$, $n = 2$ and $1/2 + 1/2$.

Let Σ be a real surface of genus g with k boundary curves $\partial_1, \cdots, \partial_k$. Let $f\colon \Sigma \longrightarrow \Sigma$ be an orientation-preserving homeomorphism which satisfies (1) there is a disjoint union of simple closed curves $\mathcal{C} = \coprod_{j=1}^{r} C_j$ such that $\mathcal{C}$ and $\partial\Sigma = \coprod_{j=1}^{k} \partial_j$ do not intersect each other, (2) $\Sigma - \mathcal{C}$ is connected and (3) $f|_{\Sigma-\mathcal{C}}$ is periodic.

Let $\widetilde{\Sigma}$ be the closed surface containing Σ obtained by pasting disks D_j ($1 \leq j \leq k$) along ∂_j and let $\widetilde{f}\colon \widetilde{\Sigma} \longrightarrow \widetilde{\Sigma}$ be the extension of f to the homeomorphism of closed surface. We can consider the natural projection $\Pi\colon \widetilde{\Sigma} \longrightarrow \widetilde{\Sigma}'$ corresponding to $\widetilde{f}$ and let g' be the genus of $\widetilde{\Sigma}'$. Let $\mathcal{A}_j$ be an annular neighborhood of C_j and let C'_j and C''_j be boundaries of A_j. We set $\mathcal{A} := \bigcup_j^r \mathcal{A}_j$. We use the following notation. We write the valency data of ∂_j by bold face characters. We enclose by the double parenthesis the valency data of the boundary curves C'_j and C''_j of $\mathcal{A}_j$. If f permutes the components of $\partial\Sigma$ or the components of $\mathcal{C}$ or the components of the boundary curves of $\mathcal{A}$, then we use the symbol of permutation. For example, if $f(\partial_1) = \partial_2$, $f(\partial_2) = \partial_3$ and $f(\partial_3) = \partial_1$, then we write $(\partial_1, \partial_2, \partial_3)$. If $C_1, \cdots, C_s$ ($s \leq r$) is amphidrome, we write $\mathrm{Amp}\{C_1, \cdots, C_s\}$. We denote the order of f by $\mathrm{ord}(f)$. We sometimes omit it if there is no fear of confusion. For each $\widetilde{f}$ which is classified in the following tables, we construct a generalized quotient $\mathcal{S}_{\widetilde{f}}$. Moreover, we add arrows at the top of the trees of $\mathcal{S}_{\widetilde{f}}$ corresponding to the center of the disks D_j (see for instance (ii2) of Table 1). We also denote this space by $\mathcal{S}_f$ and called the *marked generalized quotient space* of f. We draw the figure of marked generalized quotient space of f which are classified in the following table (ii) through (xv). Note that the sequences of integers in Table 1 mean the sequences of multiplicities of chains of nonsingular rational curves and that the thick lines mean a chain of (-2)-curves (cf. [AI]).

(i) Assume $g = 3$ and $r = k = 0$. In this case, we classify the periodic maps of curves of genus three (cf. [AI, Lemma 1.4]).

$(1) n=14,\ 11/14+5/7+1/2,$ $(2) n=14,\ 3/14+2/7+1/2,$
$(3) n=14,\ 13/14+4/7+1/2,$ $(4) n=14,\ 1/14+3/7+1/2,$
$(5) n=14,\ 9/14+6/7+1/2,$ $(6) n=14,\ 5/14+1/7+1/2,$
$(7) n=12,\ 11/12+7/12+1/2,$ $(8) n=12,\ 1/12+5/12+1/2,$
$(9) n=12,\ 11/12+3/4+1/3,$ $(10) n=12,\ 1/12+1/4+2/3,$
$(11) n=12,\ 7/12+3/4+2/3,$ $(12) n=12,\ 5/12+1/4+1/3,$
$(13) n=9,\ 8/9+4/9+2/3,$ $(14) n=9,\ 1/9+5/9+1/3,$
$(15) n=9,\ 7/9+5/9+2/3,$ $(16) n=9,\ 2/9+4/9+1/3,$
$(17) n=9,\ 8/9+7/9+1/3,$ $(18) n=9,\ 1/9+2/9+2/3,$
$(19) n=8,\ 7/8+3/8+3/4,$ $(20) n=8,\ 1/8+5/8+1/4,$
$(21) n=8,\ 5/8+5/8+3/4,$ $(22) n=8,\ 3/8+3/8+1/4,$
$(23) n=8,\ 7/8+7/8+1/4,$ $(24) n=8,\ 1/8+1/8+3/4,$
$(25) n=7,\ 6/7+6/7+2/7,$ $(26) n=7,\ 1/7+1/7+5/7,$
$(27) n=7,\ 6/7+5/7+3/7,$ $(28) n=7,\ 1/7+2/7+4/7,$
$(29) n=7,\ 6/7+4/7+4/7,$ $(30) n=7,\ 1/7+3/7+3/7,$
$(31) n=7,\ 5/7+5/7+4/7,$ $(32) n=7,\ 2/7+2/7+3/7,$
$(33) n=6,\ 5/6+1/6+1/2+1/2,$ $(34) n=6,\ 5/6+1/3+1/3+1/2,$
$(35) n=6,\ 1/6+2/3+2/3+1/2,$ $(36) n=4,\ 3/4+3/4+3/4+3/4,$
$(37) n=4,\ 1/4+1/4+1/4+1/4,$ $(38) n=4,\ 3/4+3/4+1/4+1/4,$
$(39) n=4,\ 3/4+3/4+1/2+1/2+1/2,$ $(40) n=4,\ 1/4+1/4+1/2+1/2+1/2,$
$(41) n=3,\ 2/3+2/3+2/3+2/3+1/3,$ $(42) n=3,\ 1/3+1/3+1/3+1/3+2/3,$
$(43) n=2,\quad 1/2+1/2+1/2+1/2+1/2+1/2+1/2+1/2,$
$(44) n=4,\ g'=1,\ 1/2+1/2,\quad (45) n=3,\ g'=1,\ 2/3+1/3$
$(46) n=2,\ g'=1,\ 1/2+1/2+1/2+1/2,$
$(47) n=2,\ g'=1$ and $\Pi\colon \Sigma_g \longrightarrow \Sigma_{g'}$ is an unramified covering.

(ii) Assume $g=2, r=0$ and $k=1$.

(1) $\widetilde{f}=\mathrm{id}_{\widetilde{\Sigma}}$. (2) $\mathbf{7/10}+4/5+1/2$. (3) $\mathbf{3/10}+1/5+1/2$.
(4) $\mathbf{9/10}+3/5+1/2$. (5) $\mathbf{1/10}+2/5+1/2$. (6) $\mathbf{7/8}+5/8+1/2$.
(7) $7/8+\mathbf{5/8}+1/2$. (8) $\mathbf{1/8}+3/8+1/2$. (9) $1/8+\mathbf{3/8}+1/2$.
(10) $\mathbf{5/6}+5/6+1/3$. (11) $\mathbf{1/6}+1/6+2/3$. (12) $\mathbf{4/5}+4/5+2/5$.
(13) $4/5+4/5+\mathbf{2/5}$. (14) $\mathbf{1/5}+1/5+3/5$. (15) $1/5+1/5+\mathbf{3/5}$.
(16) $\mathbf{3/5}+3/5+4/5$. (17) $3/5+3/5+\mathbf{4/5}$. (18) $\mathbf{2/5}+2/5+1/5$.
(19) $2/5+2/5+\mathbf{1/5}$. (20) $\mathbf{3/4}+1/4+1/2+1/2$. (21) $3/4+\mathbf{1/4}+1/2+1/2$.
(22) $\mathbf{2/3}+2/3+1/3+1/3$. (23) $2/3+2/3+\mathbf{1/3}+1/3$.

(24) $\mathbf{1/2} + 1/2 + 1/2 + 1/2 + 1/2 + 1/2$

(25) $g' = 1$, $\mathbf{1/2} + 1/2$.

(iii) Assume $g = 3, r = 1$ and $k = 0$. Suppose that C_1 is non-amphidrome. Then we may consider any two valencies of the fixed points of in the list of Lemma 2.3, (II) as the valencies at C_1' and C_1'' for f. Then:

(1) $\widetilde{f}|_{\Sigma-\mathcal{C}} = \mathrm{id}$. (2) $((7/8)) + ((5/8)) + 1/2$. (3) $((3/8)) + ((1/8)) + 1/2$.
(4) $((5/6)) + ((5/6)) + 1/3$. (5) $((1/6)) + ((1/6)) + 2/3$. (6) $((4/5)) + ((4/5)) + 2/5$.
(7) $((4/5)) + 4/5 + ((2/5))$. (8) $((1/5)) + ((1/5)) + 3/5$. (9) $((1/5)) + 1/5 + ((3/5))$.
(10) $((3/5)) + ((3/5)) + 4/5$. (11) $((3/5)) + 3/5 + ((4/5))$. (12) $((2/5)) + ((2/5)) + 1/5$.
(13) $((2/5)) + 2/5 + ((1/5))$. (14) $((3/4)) + ((1/4)) + 1/2 + 1/2$.
(15) $((2/3)) + ((2/3)) + 1/3 + 1/3$. (16) $((2/3)) + 2/3 + ((1/3)) + 1/3$.
(17) $((1/3)) + ((1/3)) + 2/3 + 2/3$. (18) $((1/2)) + ((1/2)) + 1/2 + 1/2 + 1/2 + 1/2$.
(19) $g' = 1$, $((1/2)) + ((1/2))$.

Suppose C_1 is amphidrome. Then:

(20) $7/10 + ((4/5)) + 1/2$. (21) $3/10 + ((1/5)) + 1/2$. (22) $9/10 + ((3/5)) + 1/2$.
(23) $1/10 + ((2/5)) + 1/2$. (24) $5/6 + 5/6 + ((1/3))$. (25) $1/6 + 1/6 + ((2/3))$.
(26) $((1/3)) + 2/3 + 1/2 + 1/2$. (27) $1/3 + ((2/3)) + 1/2 + 1/2$.
(28) $3/4 + 1/4 + ((1/2)) + 1/2$.
(29) $\Pi\colon \widetilde{\Sigma} \longrightarrow \widetilde{\Sigma}'$ is a double covering with six branch points such that the disks D_1' and D_1'' do not contain any branch points, $\widetilde{f}(C_1') = C_2'$ and the valency at C_1', C_1'' is (2, 1, 1). In this case, we write the total valency data as $1/2 + 1/2 + 1/2 + 1/2 + 1/2 + 1/2 + ((1))$. From now on, we use the same notation in a similar situation.
(30) $g' = 1$, $1/2 + 1/2 + ((1))$.

We write the figures of S_f of (iii)(1) $\sim$ (30) in (iii1) $\sim$ (iii30) of Table 1.

(iv) Assume $g = 1, r = 0$ and $k = 1$. Then:
(1) $\widetilde{f} = \mathrm{id}$. (2) $\mathbf{5/6} + 2/3 + 1/2$. (3) $\mathbf{1/6} + 1/3 + 1/2$.
(4) $\mathbf{2/3} + 2/3 + 2/3$. (5) $\mathbf{1/3} + 1/3 + 1/3$. (6) $\mathbf{3/4} + 3/4 + 1/2$.
(7) $\mathbf{1/4} + 1/4 + 1/2$. (8) $\mathbf{1/2} + 1/2 + 1/2 + 1/2$.

(v) Assume $g = 1, r = 0$ and $k = 2$.
(a) Suppose $f(\partial_i) = \partial_i$ $(i = 1, 2)$. Then:
(a1) $\widetilde{f} = \mathrm{id}$. ($a$2) $\mathbf{3/4} + \mathbf{3/4} + 1/2$. ($a$3) $\mathbf{1/4} + \mathbf{1/4} + 1/2$.
(a4) $\mathbf{2/3} + \mathbf{2/3} + 2/3$. ($a$5) $\mathbf{1/3} + \mathbf{1/3} + 1/3$. ($a$6) $\mathbf{1/2} + \mathbf{1/2} + 1/2 + 1/2$.
(b) Suppose (∂_1, ∂_2). Then:

$(b1)$ $5/6+\mathbf{2/3}+1/2$. $(b2)$ $1/6+\mathbf{1/3}+1/2$. $(b3)$ $3/4+3/4+\mathbf{1/2}$.
$(b4)$ $1/4+1/4+\mathbf{1/2}$. $(b5)$ $1/2+1/2+1/2+1/2+\mathbf{1}$.
$(b6)$ $\widetilde{f}$ is an unramified double cover.

(vi) Assume $g=1, r=0$ and $k=3$.
(a) Suppose $\widetilde{f}(\partial_i)=\partial_i$ $(i=1,2,3)$. Then:
$(a1)$ $\widetilde{f}=\mathrm{id}$. $(a2)$ $\mathbf{2/3}+\mathbf{2/3}+\mathbf{2/3}$. $(a3)$ $\mathbf{1/3}+\mathbf{1/3}+\mathbf{1/3}$. $(a4)$ $\mathbf{1/2}+\mathbf{1/2}+\mathbf{1/2}+1/2$.
(b) Suppose (∂_1,∂_2) and $\widetilde{f}(\partial_3)=\partial_3$. Then:
$(b1)$ $\mathbf{5/6}+\mathbf{2/3}+1/2$. $(b2)$ $\mathbf{1/6}+\mathbf{1/3}+1/2$. $(b3)$ $\mathbf{3/4}+3/4+\mathbf{1/2}$.
$(b4)$ $\mathbf{1/4}+1/4+\mathbf{1/2}$. $(b5)$ $\mathbf{1/2}+1/2+1/2+1/2+\mathbf{1}$.
(c) Suppose $(\partial_1,\partial_2,\partial_3)$. Then:
$(c1)$ $5/6+2/3+\mathbf{1/2}$. $(c2)$ $1/6+1/3+\mathbf{1/2}$. $(c3)$ $2/3+2/3+2/3+\mathbf{1}$. $(c4)$ $1/3+1/3+1/3+\mathbf{1}$.
$(c5)$ f comes from an unramified triple covering.

(vii) Assume $g=2, r=1$ and $k=1$.
Suppose C_1 is non-amphidrome. Then;
(1) $\widetilde{f}|_{\Sigma-\mathcal{C}}=\mathrm{id}$. (2) $((2/3))+((2/3))+\mathbf{2/3}$. (3) $((1/3))+((1/3))+\mathbf{1/3}$.
(4) $((1/2))+((1/2))+\mathbf{1/2}+1/2$.
Suppose $\mathrm{Amp}\{C_1\}$. Then:
(5) $\mathbf{5/6}+((2/3))+1/2$. (6) $\mathbf{1/6}+((1/3))+1/2$. (7) $\mathbf{3/4}+3/4+((1/2))$.
(8) $\mathbf{1/4}+1/4+((1/2))$. (9) $\mathbf{1/2}+1/2+1/2+1/2+((1))$.

(viii) Assume $g=3, r=2$ and $k=0$. Then;
(1) $\widetilde{f}|_{\Sigma-\mathcal{C}}=\mathrm{id}$. (2) $((3/4))+((3/4))+((1/2))$. (3) $((1/4))+((1/4))+((1/2))$.
(4) $((1/2))+((1/2))+((1/2))+((1/2))$. (5) $((1/2))+((1/2))+1/2+1/2+((1))$.
(6) $1/2+1/2+1/2+1/2+((1))+((1))$
(7) $\Pi\colon\widetilde{\Sigma}\to\widetilde{\Sigma}$ is an unramified double covering. $\mathrm{Amp}\{C_1), C_2\}$.
(8) (C_1,C_2), $1/2+1/2+1/2+1/2+((1))+((1))$.
(9) (C_1,C_2), $\mathrm{Amp}\{C_1,C_2\}$, $1/4+1/4+1/2+((1))+((1))$.
(10) (C_1,C_2), $\mathrm{Amp}\{C_1,C_2\}$, $3/4+3/4+1/2+((1))+((1))$.
(11) $\Pi\colon\widetilde{\Sigma}\to\widetilde{\Sigma}$ is an unramified double covering. (C_1,C_2).
(12) $\mathrm{Amp}\{C_1,C_2\}$, (C_1,C_2) and f comes from an unramified four-fold covering.

(ix) Assume $g=r=0$ and $k=3$. Then:
(a) $\widetilde{f}=\mathrm{id}$.
(b) $\mathrm{ord}(f)=2$, (∂_1,∂_2), $f(\partial_3)=\partial_3$, $\mathbf{1/2}+1/2+\mathbf{1}$.
(c) $\mathrm{ord}(f)=3$, $(\partial_1,\partial_2,\partial_3)$, $1/3+2/3+\mathbf{1}$.

(x) Assume $g = r = 0$ and $k = 4$. Then:

(a) $\widetilde{f} = \mathrm{id}$.

(b) $\mathrm{ord}(\widetilde{f}) = 2$, (∂_1, ∂_2), $f(\partial_i) = \partial_i$ $(i = 3, 4)$, $\mathbf{1/2} + \mathbf{1/2} + \mathbf{1}$.

(c) $\mathrm{ord}(\widetilde{f}) = 2$, $(\partial_1, \partial_2), (\partial_3, \partial_4)$, $1/2 + 1/2 + \mathbf{1} + \mathbf{1}$.

$(d1)$ $\mathrm{ord}(\widetilde{f}) = 3$, $(\partial_1, \partial_2, \partial_3)$, $f(\partial_4) = \partial_4$, $\mathbf{2/3} + 1/3 + \mathbf{1}$.

$(d2)$ $\mathrm{ord}(\widetilde{f}) = 3$, $(\partial_1, \partial_2, \partial_3)$, $f(\partial_4) = \partial_4$, $2/3 + \mathbf{1/3} + \mathbf{1}$.

(e) $\mathrm{ord}(\widetilde{f}) = 4$, $(\partial_1, \partial_2, \partial_3, \partial_4)$, $3/4 + 1/4 + \mathbf{1}$.

(xi) Assume $g = r = 1$ and $k = 1$. Then:

(1) $\widetilde{f}|_{\Sigma - \mathcal{C}} = \mathrm{id}$.

(2) $\mathrm{ord}(\widetilde{f}) = 2$, $\mathrm{Amp}\{C_1\}$, $\mathbf{1/2} + 1/2 + ((1))$.

(xii) Assume $g = r = 1$ and $k = 2$. Then;

$(a1)$ $\widetilde{f}|_{\Sigma - \mathcal{C}} = \mathrm{id}$.

$(a2)$ $\mathrm{ord}(\widetilde{f}) = 2$, $\mathrm{Amp}\{C_1\}$, $\mathbf{1/2} + \mathbf{1/2} + ((1))$.

$(b1)$ $\mathrm{ord}(\widetilde{f}) = 2$, $\mathrm{Amp}\{C_1\}$, (∂_1, ∂_2), $1/2 + 1/2 + ((1)) + \mathbf{1}$.

$(b2)$ $\mathrm{ord}(\widetilde{f}) = 2$, (∂_1, ∂_2), $((1/2)) + ((1/2)) + \mathbf{1}$.

(xiii) Assume $g = r = 1$ and $k = 3$. Then:

(a) $\widetilde{f}|_{\Sigma - \mathcal{C}} = \mathrm{id}$.

(b) $\mathrm{ord}(\widetilde{f}) = 2$, $\mathrm{Amp}\{C_1\}$, $\mathbf{1/2} + 1/2 + \mathbf{1} + ((1))$.

(c) $\mathrm{ord}(\widetilde{f}) = 3$, $(\partial_1, \partial_2, \partial_3)$, $((1/3)) + ((2/3)) + \mathbf{1}$.

(xiv) Assume $g = r = 2$ and $k = 1$. Then:

(1) $\widetilde{f}|_{\Sigma - \mathcal{C}} = \mathrm{id}$.

(2) $\mathrm{ord}(\widetilde{f}) = 2$, $\mathrm{Amp}\{C_1, C_2\}$, $\mathbf{1/2} + 1/2 + ((1)) + ((1))$.

(3) $\mathrm{ord}(\widetilde{f}) = 2$, (C_1, C_2), $\mathbf{1/2} + 1/2 + ((1)) + ((1))$.

(4) $\mathrm{ord}(\widetilde{f}) = 4$, $\mathrm{Amp}\{C_1, C_2\}$, $(C_1', C_2', C_1'', C_2'')$, $\mathbf{3/4} + 1/4 + ((1))$.

(5) $\mathrm{ord}(\widetilde{f}) = 4$, $\mathrm{Amp}\{C_1, C_2\}$, $(C_1', C_2', C_1'', C_2'')$, $3/4 + \mathbf{1/4} + ((1))$.

(xv) Assume $g = r = 3$ and $k = 0$. Then:

(1) $\widetilde{f}|_{\Sigma - \mathcal{C}} = \mathrm{id}$.

(2) $\mathrm{ord}(\widetilde{f}) = 2$, $\mathrm{Amp}\{C_1, C_2\}$, $f(C_3) = C_3$, $((1/2)) + ((1/2)) + ((1)) + ((1))$.

(3) $\mathrm{ord}(\widetilde{f}) = 2$, $\mathrm{Amp}\{C_1, C_2, C_3\}$, $1/2 + 1/2 + ((1)) + ((1)) + ((1))$.

(4) $\mathrm{ord}(\widetilde{f}) = 2$, $\mathrm{Amp}\{C_3\}$, (C_1, C_2) $1/2 + 1/2 + ((1)) + ((1)) + ((1))$.

(5) $\mathrm{ord}(\widetilde{f}) = 2$, (C_1, C_2), $f(C_3) = C_3$, $((1/2)) + ((1/2)) + ((1)) + ((1))$.

(6) $\mathrm{ord}(\widetilde{f}) = 3$, (C_1, C_2, C_3), $1/3 + 2/3 + ((1))$.

(7) $\mathrm{ord}(\widetilde{f}) = 4$, $\mathrm{Amp}\{C_1, C_2\}$, $\widetilde{f}(C_3) = C_3$, $(C_1', C_2', C_1'', C_2'')$, $((1/4)) + ((3/4)) + ((1))$.
(8) $\mathrm{ord}(\widetilde{f}) = 6$, $\mathrm{Amp}\{C_1, C_2, C_3\}$, $(C_1', C_2', C_3', C_1'', C_2'', C_3'')$, $1/6 + 5/6 + ((1))$.

2.3 Monodromies and their generalized quotients

For Step 4, we introduce the definition of the quotient graphs and the notion of "substitution"

Let X and $G = \langle \sigma \rangle$ be a weighted oriented graph and a cyclic automorphism group of X described in Lemma 2.2. We define the quotient graph Y of X with respect to G as follows (cf. [MM1, §7]): Y is a weighted graph which may have loops and satisfies the following properties:

(i) There exists a map $h: X \longrightarrow Y$ of graphs.

(ii) Let $|X|$ and $|Y|$ be the underlying 1-dimensional cell complex of X and Y, respectively. Then the map h naturally induces a finite covering map $|h|: |X| \longrightarrow |Y|$ such that the covering transformation group of $|h|$ coincides with G.

(iii) Let $\bar{v}$ be a vertex of Y. Then $h^{-1}(\bar{v})$ consists of a finite number, say $l(\bar{v})$, of vertices v_i $(1 \leq i \leq l(\bar{v}))$ such that their weights $(g(v_i), \rho(v_i))$ coincide with each other, and denoted by $(g(\bar{v}), \rho(\bar{v}))$. In this sense, $\bar{v}$ has the triple weight $(l(\bar{v}), g(\bar{v}), \rho(\bar{v}))$.

(iv) Let $\bar{e}$ be an edge of Y. Then $h^{-1}(\bar{e})$ consists of a finite number, say $\xi(\bar{e})$ of edges of X. We put $\xi(\bar{e})$ on $\bar{e}$ as the weight.

We remark that, if the vertices v, v' which are ends of an edge e satisfies $h(v) = h(v')$, then $h(e)$ is a loop of Y starting from and ending at $h(v)$.

Next we define the resolution $\widetilde{Y}$ of Y. Suppose that there exists an edge e of X and an positive integer m which satisfy $\sigma^m(v) = v'$ and $\sigma^m(v') = v$, where v and v' are the vertices at both ends of e. Then we replace $\sigma(e)$ by a line with weight $2m$ where the top part is branched into two lines with weight m (see for instance Graph(6) in Table 3). We call the sum of these three lines *D-edge* of weight $2\overline{m}$. After completing this process for every edge with the above property, we obtain a weighted graph $\widetilde{Y}$, which we denote by $\widetilde{Y} = X/G$.

In Table 3 , the number inside a small circle (i.e., a vertex $\bar{v}$) means $g(\bar{v}) + \rho(\bar{v})$, and the number beside the circle means $l(\bar{v})$. The number beside an edge $\bar{e}$ means $\xi(\bar{e})$. A loop is written with two arrows (see for instance Graph(5)). If $g(\bar{v}) + \rho(\bar{v}) = 0$ or $l(\bar{v}) = 1$ or $\xi(\bar{e}) = 1$, then it is omitted. Note that one graph might represent several weighted graphs $\widetilde{Y}$ as in the case of Table 2. Then we have;

Lemma 2.4 [AI, Lemma 3.6] *For each weighted oriented graph X and each automorphism group G of X in Lemma* 2.2*, the resolution X/G of quotient graph are as in* Table 2.

For a graph $\widetilde{Y} = X/G$, we introduce the notion of *substitution of marked generalized quotient* in the following way: Since $\widetilde{Y}$ is planer, we have an embedding $\iota\colon \widetilde{Y} \hookrightarrow E^2$ into Euclidean plane E^2. We fix ι. Let v be a vertex of $\widetilde{Y}$. Let $B(v,\varepsilon)$ be a closed ball of small radius ε in E^2 with center v, and we set

$$V = \widetilde{Y} \cap B(v,\varepsilon).$$

Suppose $e_1,\dots,e_s,\ e_{s+1},\dots,e_{s+s'}$ are the edges of $\widetilde{Y}$ containing v as end with each of $e_{s+1},\dots,e_{s+s'}$ being a loop. Then V consists of a vertex v and $s+2s'$ segments $e'_1,\dots,e'_s$, $e'_{s+1}, e''_{s+1}\dots, e'_{s+s'}, e''_{s+s'}$ ($e_i \cap B(v,\varepsilon) = e'_i$ for $1 \le i \le s$, $e_i \cap B(v,\varepsilon) = e'_i \cup e''_i$ for $s+1 \le i \le s+s'$). Moreover, V has a natural weighted graph structure induced by $\widetilde{Y}$, i.e., the vertex v has triple weight $(l(v), g(v), \rho(v))$ and the edge e'_i ($1 \le i \le s+s'$) has weight $\xi(e_i)$, which the edge e''_i ($s+1 \le i \le s+s'$) has weight $\xi(e_i)$.

On the other hand, let Σ be a surface of genus at most three with k boundaries. Let $f\colon \Sigma \longrightarrow \Sigma$ be one of the pseudo-periodic maps whose admissible system consists of r curves classified in § 2.2, and satisfies the following conditions:
(i) $g = g(v)$, $r = \rho(v)$ and $k = (\sum_{i=1}^{s+s'} \xi(e'_i) + \sum_{i=s+1}^{s+s'} \xi(e''_i))/l(v)$.
(ii) $\mathcal{S}_f$ has $s+2s'$ arrows. Set $\mathcal{S}_f = \sum_j m_j E_j + \sum_i n_i \overrightarrow{F}_i$ where E_j is a component of $\mathcal{S}_f$ and $\overrightarrow{F}_i$ is an arrow of $\mathcal{S}_f$ (m_i, m_j are their multiplicities). Changing the order if necessary, we have

$$\xi(e'_i) = l(v)n_i\ (1 \le i \le s+s'),\ \xi(e''_i) = l(v)n_i\ (s+s'+1 \le i \le s+2s').$$

We denote by $E(\overrightarrow{F}_i)$ the component of $\sum E_j$ which intersect $\overrightarrow{F}_i$. We substitute $\mathcal{S}_f$ for V in $\widetilde{Y}$ in the following way: we replace the vertex v by $\sum_j l(v)m_jE_j$ and connect each edge e'_i (or e''_i) to $\sum_j l(v)m_jE_j$ so that e'_i (or e''_i) intersect $E(\overrightarrow{F}_i)$ transversally.

We perform this process for each part V of $\widetilde{Y}$, and replace each edge e of $\widetilde{Y}$ by trees of spheres. We also substitute each D-edge by a tree of spheres of Dynkin diagram of type D (see [MM2, p.73 Figure 3]). In this way, we obtain the generalized quotient space S_f for a certain pseudo-periodic map $f\colon \Sigma_3 \longrightarrow \Sigma_3$ of negative type. The map f has the following properties: The graph of the admissible system of cut curves of f coincides with X, the action to X of f coincides with G, and the stabilizer of G for each connected component $\mathcal{B}_i$ of $\Sigma_3 - \mathcal{C}$ coincides with one of the periodic maps in §2.2 whose marked generalized quotient space is just substituted to V_i corresponding to B_i.

For an example, let $X = K_1$ and $G = \mathrm{II}(1,4)$. Then the parts of $K_1/\mathrm{II}(1,4)$ in Table 3 (19) consist of

$$\begin{aligned} V_1 &= \{v_1, e_1'; l(v_1) = g(v_1) = 1, \rho(v_1) = 0, \xi(e_1') = 1\} \\ V_2 &= \{v_2, e_1', e_2'; l(v_2) = 1, g(v_2) = \rho(v_2) = 0, \xi(e_1') = \xi(e_2') = 1\} \\ V_3 &= \{v_3, e_2', e_3', e_3''; l(v_3) = 2, g(v_3) = \rho(v_3) = 0, \xi(e_2') = \xi(e_3') = \xi(e_3'') = 2\}. \end{aligned}$$

By our rule, we substitute (iv1) $\sim$ (iv8) in Table 1 for V_1, (ix$b$) for V_2 and (ix$a$) for V_3. We write this result as

$$K_1,\ II(1,4),\ V_1 = (\mathrm{iv}),\ V_2 = (\mathrm{ix}b),\ V_3 = (\mathrm{ix}a).$$

Now we classify the conjugacy classes of the pseudo-periodic maps of negative type of genus three. By Matsumoto-Montesinos [MM2, Theorem 2], it is equivalent to classifying triples (X, G, S_f) in our notation.

Since the replacement by (ix$a$) or (xa) is trivial, we omit it in the following theorem.

Theorem 2.5 [AI, Proposition 3.8]

The conjugacy classes of the pseudo-periodic maps of negative type of genus three are classified as follows: (1) A_3 : Id, $V_1 = $ (i) in § 2.2.
(2) A_2 : Id, $V_1 = $ (iii).
(3) A_1 : Id, $V_1 = $ (viii).
(4) A_0 : Id, $V_1 = $ (xv).
(5) B_{21} : Id, $V_1 = $ (ii), $V_2 = $ (iv).
(6) B_{20} : Id, $V_1 = $ (ii), $V_2 = $ (xi).
(7) B_{11} : Id, $V_1 = $ (vii), $V_2 = $ (iv).
(8) B_{10} : Id, $V_1 = $ (vii), $V_2 = $ (xi).
(9) B_{01} : Id, $V_1 = $ (xiv), $V_2 = $ (iv).
(10) B_{00} : Id, $V_1 = $ (xiv), $V_2 = $ (xi).
(11) C_{111} : Id, $V_1 = V_2 = $ (iv), $V_3 = $ (va). II(1,1), $V_1 = $ (iv), $V_2 = $ (vb).
(12) C_{110} : *Id*, $V_1 = V_2 = $ (iv), $V_3 = $ (xiia). II(1,1), $V_1 = $ (iv), $V_2 = $ (xiib).
(13) C_{101} : Id, $V_1 = $ (iv), $V_2 = $ (xi), $V_3 = $ (va).
(14) C_{100} : Id, $V_1 = $ (iv), $V_2 = $ (xi), $V_3 = $ (xiia).
(15) C_{001} : Id, $V_1 = V_2 = $ (xi), $V_3 = $ (va). II(1,1), $V_1 = $ (xi), $V_2 = $ (vb).
(16) C_{000} : Id, $V_1 = V_2 = $ (xi), $V_3 = $ (xii$a$). II(1,1), $V_1 = $ (xi), $V_2 = $ (xii$b$).
(17) D_{111} : Id, $V_1 = V_2 = V_3 = $ (iv). II(1,1), $V_1 = V_3 = $ (iv), $V_2 = $ (xb).
III(1,1), $V_1 = (iv), V_2 = (ixc)$.

(18) D_{110} : Id, $V_1 = V_2 = (iv), V_3 = $ (xi). II(1,1), $V_1 =$ (iv)$, V_2 =$ (xb)$, V_3 =$ (xi).

(19) D_{100} : Id, $V_1 =$ (iv)$, V_2 = V_3 =$ (xi). II(1,1), $V_1 =$ (xi)$, V_2 =$ (xb)$, V_3 =$ (iv).

(20) D_{000} : Id, $V_1 = V_2 = V_3 =$ (xi). II(1,1), $V_1 = V_3 =$ (xi)$, V_2 =$ (xb).
III(1,1), $V_1 =$ (xi)$, V_2 =$ (ix$c$).

(21) E_{11} : Id, $V_1 = V_2 =$ (va). II(0,1), $V_1 = V_2 =$ (vb). II(1,2), $V_1 =$ (va).
III(1,3), $V_1 =$ (va).

(22) E_{10} : Id, $V_1 =$ (va)$, V_2 =$ (vii$a$). II(0,1), $V_1 =$ (vb)$, V_2 =$ (xii$b$).

(23) E_{00} : Id, $V_1 = V_2 =$ (vii$a$). II(0,1), $V_1 = V_2 =$ (vii$b$). II(1,2), $V_1 =$ (xii$a$).
III(1,3), $V_1 =$ (xii$a$).

(24) F_{11} : Id, $V_1 = V_2 =$ (iv). II(0,1), $V_1 = V_4 =$ (iv)$, V_2 = V_3 =$ (ix$b$).
II(2,1), $V_1 =$ (iv). II(2,2), $V_1 =$ (iv).

(25) F_{10} : Id, $V_1 =$ (iv)$, V_2 =$ (xi). II(0,1), $V_1 =$ (iv)$, V_2 = V_3 =$ (ix$b$)$, V_4 =$ (xi).

(26) F_{00} : Id, $V_1 = V_2 =$ (xi). II(0,1), $V_1 = V_4 =$ (xi)$, V_2 = V_3 =$ (ix$b$).
II(2,1), $V_1 =$ (xi). II(2,2), $V_1 =$ (xi).

(27) G_{11} : Id, $V_1 =$ (va)$, V_2 =$ (iv). II(0,1), $V_1 =$ (vb)$, V_2 =$ (ix$b$)$, V_3 =$ (iv).

(28) G_{10} : Id, $V_1 =$ (va)$, V_2 =$ (xi). II(0,1), $V_1 =$ (vb)$, V_2 =$ (ix$b$)$, V_3 =$ (xi).

(29) G_{01} : Id, $V_1 =$ (xii$a$)$, V_2 =$ (iv). II(0,1), $V_1 =$ (xii$b$)$, V_2 =$ (ix$b$)$, V_3 =$ (iv).

(30) G_{00} : Id, $V_1 =$ (xii$a$)$, V_2 =$ (xi). II(0,1), $V_1 =$ (vii$b$)$, V_2 =$ (ix$b$)$, V_3 =$ (xi).

(31) H_1 : Id, $V_1 =$ (vi$a$). II(0,1), $V_1 =$ (vi$b$)$, V_2 =$ (ix$b$).
III(0,1), $V_1 =$ (vi$c$)$, V_2 =$ (ix$c$).

(32) H_0 : Id, $V_1 =$ (xiii$a$). II(0,1), $V_1 =$ (xiii$b$)$, V_2 =$ (ix$b$).
III(0,1), $V_1 =$ (xiii$c$)$, V_2 =$ (ix$c$).

(33) I_1 : Id, $V_1 =$ (iv). II(0,1), $V_1 =$ (ix$b$)$, V_2 =$ (xb)$, V_3 =$ (iv).
III(0,1), $V_1 =$ (ix$c$)$, V_2 =$ (xd)$, V_3 =$ (iv).

(34) I_0 : Id, $V_1 =$ (xi). II(0,1), $V_1 =$ (ix$b$)$, V_2 =$ (xb)$, V_3 =$ (xi).
III(0,1), $V_1 =$ (ix$c$)$, V_2 =$ (xd)$, V_3 =$ (xi).

(35) J_1 : Id, $V_3 =$ (va). II(0,1), $V_1 =$ (va)$, V_2 = V_3 =$ (ix$b$).
II(1,4), $V_1 =$ (vb). II(1,6), $V_1 =$ (vb).

(36) J_0 : Id $V_3 =$ (xii$a$). II(0,1), $V_1 =$ (xii$a$)$, V_2 = V_3 =$ (ix$b$).
II(1,4), $V_1 =$ (xii$b$). II(1,6), $V_1 =$ (xii$b$).

(37) K_1 : Id, $V_4 =$ (vi). II(0,1), $V_1 = V_2 =$ (ix$b$)$, V_4 =$ (iv).
II(1,4), $V_1 =$ (iv)$, V_2 =$ (ix$b$). II(1,6), $V_1 =$ (iv)$, V_2 =$ (ix$b$).

(38) K_0 : Id, $V_4 =$ (xi). II(0,1), $V_1 = V_2 =$ (ix$b$)$, V_4 =$ (xi).
II(1,4), $V_1 =$ (xi)$, V_2 =$ (ix$b$). II(1,6), $V_1 =$ (xi)$, V_2 =$ (ix$b$).

(39) L : Id. II(0,1), $V_1 = V_2 =$ (xb). II(0,2), $V_1 = V_2 =$ (xc). II(1,5).
II(1,7). II(1,8). III(0,1), $V_1 = V_2 =$ (xd). IV(0,1), $V_1 = V_2 =$ (xe).
IV(1,1), $V_1 =$ (xc). VI(1,1), $V_1 =$ (xd).

(40) M : Id. II(0,1), $V_1 =$ (ix$b$), $V_3 =$ (xb). II(0,2), $V_1 =$ (xc), $V_2 = V_3 =$ (ix$b$).
II(1,9), $V_1 =$ (xc). IV(1,2), $V_1 =$ (xe), $V_2 =$ (ix$b$).

(41) N : Id. II(0,1), $V_1 = V_3 =$ (ix$b$). II(0,2), $V_1 = V_2 = V_3 = V_4 =$ (ix$b$).
IV(0,1). II(2,4). II(2,5). II(2,7). II(2,8).
IV(2,1), $V_1 = V_2 =$ (ix$b$). IV(2,2), $V_1 = V_2 =$ (ix$b$).

(42) O : Id. II(1,10), $V_1 = V_2 =$ (ix$b$). II(2,6). III(1,2), $V_1 =$ (ix$c$). IV(1,3).

3 Possibility of the existence.

3.1 Non-periodic case

First, we introduce Horikawa's canonical resolution of singularities which appear in double coverings of a surface (cf. [Ho1, §2]).

Let (W_0, B_0) be a pair of nonsingular surface and its divisor which is free from multiple components. We sometimes regard B_0 as a line bundle on W_0. Assume that there exists a line bundle F_0 on W_0 such that $B_0 \simeq F_0^{\otimes 2}$. Let $\{U_i\}$ be a finite open covering of W_0 and let $b_i = 0$ and $\{f_{ij}\}$ are local defining equation of B_0 on each $\{U_i\}$ and a system of transition functions of F_0, respectively. We may assume that $b_i = f_{ij}^2 b_j$ on $U_i \cap U_j$. Let w_i denote fiber coordinates of the line bundle F_0 over U_i and define a subvariety S_0 of F_0 whose defining equations are $w_i^2 - b_i = 0$. This is well-defined because $w_i^2 - b_i = f_{ij}^2(w_j^2 - b_j)$ over $U_i \cap U_j$. Since B_0 has no multiple component, S_0 is a normal variety. Moreover, if B_0 is nonsingular, S_0 is also nonsingular. We shall call S_0 the *double covering* with branch locus B_0. Let $\tau_1 \colon W_1 \longrightarrow W_0$ be a blowing-up with center at a singular point P_1 of B_0 and let E_1 be the exceptional divisor of τ_1 which we sometimes regard as a line bundle on W_1. We set $B_1 = \tau_1^* B_0 - 2[m_1/2]E_1$ and $F_1 = \tau^* F_0 - [m_1/2]E_1$, where m_1 is the multiplicity of B_0 at P_1 and $[m_1/2]$ denotes the greatest integer not exceeding $m_1/2$. Since we have linear equivalence $B_1 \sim F_1^{\otimes 2}$, there exists a double covering S_1 of W_1 with branch locus B_1. Letting U_1 be a coordinate neighborhood on W_0 whose center is P_1, we may assume that $w_1^2 = b_1$ is the defining equation of S_0 over U_1.

$\tau_1^{-1}(U_1)$ is covered by a finite number of coordinate neighborhoods $\{V_i\}$ on W_1 and S_1 is defined by the equation $\widetilde{w}_i^2 = \widetilde{b}_i$ over each V_i, where $\widetilde{w}_i$ and $\widetilde{b}_i$ are the fiber coordinate on F_1 and the equation of B_1 on V_1, respectively. Let e_i be the equation of E_1 on V_i.

Then we can define a birational holomorphic map $S_1 \longrightarrow S_0$ by

$$(z, w_i) \mapsto (q_1(z), e_i^{[m_1/2]} w_i) \in U_1.$$

Since a singular curve is resolved by a finite blowing-ups, we obtain a nonsingular model S_n after a finite number of above process. We call S_n the *canonical resolution* of S_0.

We apply above process to a hyperelliptic family of genus three. Let $\phi\colon S \longrightarrow \Delta$ be a normally minimal hyperelliptic family of genus g. By the same argument in [Ho2, §1], we see that S is bimeromorphic to a double covering $\psi_0\colon S_0 \longrightarrow W_0 := \mathbf{P}^1 \times \Delta$ branched along a divisor B_0 of W_0. More precisely, there is a line bundle F on $\mathbf{P}^1 \times \Delta$ which satisfies $B_0 \sim F^{\otimes 2}$ and S_0 is realized in F as a double covering of $\mathbf{P}^1 \times \Delta$. Let π_0 be the second projection of W_0. We set $\Gamma_t = \pi_0^{-1}(t)$, $\widetilde{B}_0 := B_0 - \Gamma_0$ when Γ_0 is a component of B_0, and $\widetilde{B}_0 := B_0$ otherwise. (S_0, B_0) satisfies the following conditions:

(i) $(B_0.\, \Gamma_t) = 2g+2$ for $t \in \Delta$, where $(B_0.\, \Gamma_t)$ is the intersection number of B_0 and Γ_t.

(ii) If a point P satisfies $I_P(\widetilde{B}_0, \pi_0^{-1}(t)) \geq 2$, then P is on Γ_0, where $I_P(\widetilde{B}_0, \Gamma_t)$ is the local intersection number of $\widetilde{B}_0$ and Γ_t at P.

Let $\tau_1\colon W_1 \longrightarrow W_0$ be the blowing-up at a point P which satisfies $I_P(B_0, \Gamma_0) \geq 2$ and let $\pi_1\colon W_1 \longrightarrow \Delta$ be the composite $\pi_0 \circ \tau_1$. We set $E_1 := \tau_1^{-1}(P)$, $B_1 := \tau_1^*(B_0) - 2[m_P/2]E_1$ and $F_1 := \tau_1^* F_0 - [m_p/2]E_1$, where m_P is the multiplicity of B_0 at P and $[m_P/2]$ is the greatest integer not exceeding $m_P/2$. Since $[B_1] = 2F_1$, there exists a double covering $\psi_1\colon S_1 \longrightarrow W_1$ branched along B_1. By the same argument in [Ho1, §2], we can construct a bimeromorphic map $\widetilde{\tau}_1\colon S_1 \longrightarrow S_0$ which satisfies $\psi_0 \circ \widetilde{\tau}_1 = \tau_1 \circ \psi_1$ and $\pi_0 \circ \psi_0 \circ \widetilde{\tau}_1 = \pi_1 \circ \psi_1$. We call a point P on B_i a *bad point* if B_i is singular at P or $I_P((\tau_1 \circ \cdots \tau_i)^*(\Gamma_0)_{\mathrm{red}}, \widetilde{B}_i) \geq 2$, where $\widetilde{B}_i$ is the strict transform of $\widetilde{B}_0$ by $\tau_1 \circ \cdots \circ \tau_i$. Repeating this process at bad points, we obtain a sequence of blowing-ups $W_r \xrightarrow{\tau_r} \cdots \longrightarrow W_1 \xrightarrow{\tau_1} W_0$ which satisfies the following properties:

(i) $(B_r)_{\mathrm{red}}$ is nonsingular.

(ii) $\Theta := (\tau_1 \circ \cdots \circ \tau_r)^*(\Gamma_0)$ and the strict transform of $\widetilde{\widetilde{B}_0}$ intersect each other transversally.

S_r is nonsingular by (i). The reduced scheme of the special fiber of S_r is a normal crossing divisor by (ii) and $\phi\colon S \longrightarrow \Delta$ is the normally minimal model of $\phi_r = \pi_r \circ \psi_r\colon S_r \longrightarrow \Delta$ which satisfies $\phi \circ \widetilde{\tau} = \phi_r$, where $\widetilde{\tau}$ is the composite of blowing-downs of suitable (-1)-curves. We call above process *Horikawa's canonical resolution.*

Lemma 3.1 *Let E be a component of $(\tau_1 \circ \cdots \circ \tau_i)^*(\Gamma_0)$ whose multiplicity α is greater than or equal to 2. Assume that E intersects three different components E_{j_1}, E_{j_2}, E_{j_3} of $(\tau_1 \circ \cdots \circ \tau_i)^*(\Gamma_0)$. Let $\widehat{E}_{j_i}$ $(i = 1, 2, 3)$ be the maximal connected subdivisors of $(\tau_1 \circ \cdots \circ \tau_i)^*(\Gamma_0)$ such that their supports do not contain E and that $\widehat{E}_{j_i} \geq E_{j_i}$. If α does not divide the greatest common divisor of the multiplicities of the components of $\widehat{E}_{j_i}$ $(i = 1, 2)$, then there exists a subdivisor D of $(\tau_1 \circ \cdots \circ \tau_i)^*(\Gamma_0)$ such that $\widehat{E}_{j_3}$ is equal to αD.*

Proof If there exists no divisor D such that $\widehat{E}_{j_3}$ is equal to αD, then there exists r' $(1 \leq r' \leq r)$ such that three different exceptional sets of $\tau_1 \circ \cdots \circ \tau_{r'}$ meet at a point, a contradiction to the process of Horikawa's canonical resolution. q.e.d.

Let C be a prime divisor of S which is a component of $\phi^{-1}(0)$ and let Z be the set of points which are the images of (-1)-curves of $\widetilde{\tau}$. We let $\Pi(C) := \overline{\psi_r \circ \widetilde{\tau}^{-1}(C - Z)}$ be the closure of $\psi_r \circ \widetilde{\tau}^{-1}(C - Z)$ in W_r. We also see $\Pi(C)$ as a prime divisor on W_r.

Let C be a prime divisor of S which is a component of $\phi^{-1}(0)$. Assume C' to be another component of $\phi^{-1}(0)$ which satisfies $\Pi(\mathrm{C}) \neq \Pi(\mathrm{C}')$ and $\Pi(\mathrm{C})$ intersects $\Pi(\mathrm{C}')$ at a point. Since the dual graph of Θ is connected, there exists a subdivisor $D_{CC'} = \sum a_i E_i$ of Θ which satisfies the following conditions:

(i) $\Theta \geq D_{CC'}$ and $\Theta \not\geq D_{CC'} + E_i$ for all E_i $(a_i \neq 0)$.

(ii) $\mathrm{Supp}(\Pi(C)) \cap D_{CC'} \neq \emptyset$ and $\mathrm{Supp}(\Pi(C')) \cap D_{CC'} \neq \emptyset$.

(iii) $D_{CC'} \not\geq \Pi(C)$ and $D_{CC'} \not\geq \Pi(C')$.

(iv) $\mathrm{Supp}(D_{CC'})$ is connected.

We call the divisor $D_{CC'}$ the *bridge* between $\Pi(C)$ and $\Pi(C')$. Since the dual graph of Θ has no loop, the bridge is uniquely determined.

By semistable reduction theorem (cf. [DM]), for any degeneration (ϕ, S, Δ), there exists a branched cover $\Delta' \longrightarrow \Delta$ totally ramified over 0 such that the minimal resolution S' of $S \times_\Delta \Delta' \longrightarrow \Delta'$ is a semistable family. We call $S' \longrightarrow \Delta'$ the *semistable model* of (ϕ, S, Δ). Let $S' \longrightarrow S''$ be a composite of blowing-downs of (-2)-curves and assume S'' to be free from (-2)-curves. We call $S'' \longrightarrow \Delta'$ the *stable model* of (ϕ, S, Δ). We sometimes call the special fiber of the (semi)stable model of (ϕ, S, Δ) the *(semi)stable model.* Let F_{ss} be a semistable curve. If there exists no hyperelliptic family whose special fiber is F_{ss}, there neither exists a hyperelliptic family whose semistable model has F_{ss} as the special fiber, because if (ϕ, S, Δ) is a hyperelliptic family, the (semi)stable model is also a hyperelliptic family.

Figure (A) to (O) in Table 2 can be regarded as the dual graphs of stable curves of genus three. Furthermore, if we replace each edge of these graphs by a chain of (-2)-curves, these graphs can be regarded as the weighted dual graphs of semistable curves of genus three. We call a chain of (-2)-curves a $\mathbf{P}^1$-*chain* at the edge and call the number of components of a $\mathbf{P}^1$-chain the *length* of the $\mathbf{P}^1$-chain.

Theorem 3.2 *There exists no hyperelliptic family of genus three whose topological type of the special fiber of the semistable model is* (D), (H), (I), (M), (O) *in* Table 2.

PROOF We use the same notation as in the above paragraph. Since the degeneration which is obtained by a base change of a hyperelliptic family is a hyperelliptic family, we may assume the hyperelliptic family $\phi: S \longrightarrow \Delta$ to be semistable. Moreover, we only consider the case of a stable family, because the case of a semistable family is similar. The vertices of the graphs are regarded as the corresponding irreducible curve for the simplicity of the notation.

Case (D). Note that $\Pi(v_i)$ $(1 \leq i \leq 4)$ are not components of the branch locus, because the multiplicity of each v_i is one. Since v_i $(1 \leq i \leq 3)$ intersects v_4 at a point, the bridge between $\Pi(v_i)$ and $\Pi(v_4)$ intersects v_4 at one point P_i, and P_i is contained within the branch locus of ψ_r. It means that v_4 is the double covering of $\mathbf{P}^1$ branched at least at three points. This contradicts the fact that v_4 is $\mathbf{P}^1$.

Case (H), (I). $\Pi(v_1)$ is not equal to $\Pi(v_2)$, because v_1 is not homeomorphic to v_2 in the case of (H) and v_2 intersects v_3 in the case of (I). Since v_1 intersects v_2 at three points, there exists at least two bridges between $\Pi(v_1)$ and $\Pi(v_2)$, a contradiction.

Case (M). If $\Pi(v_1) \neq \Pi(v_2)$, the dual graph of Θ has a loop. We may assume $\Pi(v_1) = \Pi(v_2)$. Since $\Pi(v_1) \neq \Pi(v_3)$, there exists at least two bridges between $\Pi(v_3)$ and $\Pi(v_1) = \Pi(v_2)$, a contradiction.

Case (O). We may assume $\Pi(v_1) = \Pi(v_2)$ and $\Pi(v_3) = \Pi(v_4)$. In view of the configuration of (O), there exists at least two bridges between $\Pi(v_1)$ and $\Pi(v_3)$, a contradiction. q.e.d.

Theorem 3.3 *There exist (semi)stable families of hyperelliptic families whose topological types of the special fibers are* (A), (B), (C), (E), (F), (G), (J), (K), (L), (N), *if the following conditions are satisfied:*

(i) *The length of the* $\mathbf{P}^1$*-chains are the same at* e_1 *and* e_2 *in the cases of* (E), (F) *and* (G).

(ii) *The length of the* $\mathbf{P}^1$*-chains are the same at* e_3 *and* e_4 *in the cases of* (F), (J) *and* (K).

(iii) *The length of the* $\mathbf{P}^1$*-chains are the same at* e_5 *and* e_6 *in the case of* (N).

There exist no hyperelliptic families if the above conditions are not satisfied.

PROOF We prove the existence of the families in Section 4 by giving explicit examples of the equations of the double coverings.

If there exists a family whose special fiber is (E) and which does not satisfy the above condition (i), then there exist at least two bridges between $\Pi(v_1)$ and $\Pi(v_2)$, a contradiction. The other cases are proved by the same argument. q.e.d.

In the following theorem, a screw number is said to be "special" if no example of equation of the monodromies appears in Section 4.

Theorem 3.4 *There exist no hyperelliptic families whose topological monodromies are the following:*

A_2: (iii2), (iii3), (iii7), (iii9), (iii11), (iii13), (iii14), (iii16), (iii18), (iii28).

A_1: (viii5), (viii12).

A_0: (xv2), (xv7).

B_{1i} $(i=0,1)$: $V_1 = \{(\text{vii4}), (\text{vii7}), (\text{vii8})\}$.

B_{0i} $(i=0,1)$: $V_1 = (\text{xiv3})$.

C_{111}: Id, $V_3 = \{(\text{va4}), (\text{va5})\}$.

C_{111}: II(1,1) $V_3 = \{(\text{vb1}), (\text{vb2})\}$.

C_{101}: Id, $V_3 = \{(\text{va4}), (\text{va6})\}$.

C_{001}: Id, $V_3 = \{(\text{va4}), (\text{va6})\}$.

C_{001}: II(1,1), $V_3 = \{(\text{vb1}), (\text{vb2})\}$.

N: II(2,5), II(2,7), IV(2,1).

E_{ii} $(i=0,1)$: II(0,1), *and the screw number at* e_1 *is special.*

F_{ii} $(i=0,1)$: II(0,1), *and the screw number at* e_1 *is special.*

G_{ij} $(i=0,1)$: II(0,1), *and the screw number at* e_1 *is special.*

J_i $(i=0,1)$: II(1,4), *and the screw number at* e_1 *is special.*

K_i $(i=0,1)$: II(1,4), *and the screw number at* e_1 *is special.*

PROOF Since there are too many cases, we write down the proof for only five typical cases.

Case A_2: (iii2). Assume that there exists a normally minimal hyperelliptic family whose topological monodromy is A_2: (iii2). The dual graph of its special fiber has a loop, and the multiplicities of components of the loop are not the same. Hence there exists a loop in the dual graph of Θ, a contradiction. The cases of A_2: (iii3), (iii7), (iii9), (iii11), (iii13), (iii14), (iii16), C_{111}: Id, $V_3 = (\mathrm{va}4, 5)$, C_{111}: II(1,1), $V_3 = (\mathrm{vb}1, 2)$, C_{101}: Id, $V_3 = (\mathrm{va}4, 6)$, C_{001}: Id, $V_3 = (\mathrm{va}4, 6)$, C_{001}: II(1,1), $V_3 = (\mathrm{vb}1, 2)$, N: IV(2,1), can be proved by the same argument.

Case A_2: (iii18). If there exists a hyperelliptic family whose topological monodromy is A: (iii18), then we have a contradiction to Lemma 3.1. The cases of A_2: (iii28), A_1: (viii5), A_0: (xv2), (xv7), B_{1i} $(i = 0, 1)$: $V_1 = \{(\mathrm{vii}4), (\mathrm{vii}7), (\mathrm{vii}8)\}$, B_{0i} $(i = 0, 1)$: $V_1 = (\mathrm{xiv}3)$, N: II(2,7), can be proved by the same argument.

Case A_1: (viii12). If there exists a hyperelliptic family S whose monodromy is A_1: (viii12), the configuration of the special fiber is as shown in Figure 1. We consider $\psi_0 \colon S_0 \longrightarrow \Delta \times \mathbf{P}^1$ as in §3.1. Since the greatest common divisor of multiplicities of the components of the special fiber is equal to two, Γ_0 has to be a component of the branch locus of ψ_0. Let $\widetilde{\Gamma}_0$ be the strict transform of Γ_0 in W_r. Assume that $\Pi(v_1)$ is not a component of the branch locus. Then $\Pi(v_1)$ intersects the bridge between $\Pi(v_1)$ and $\Pi(v_4)$ at a point which is contained in the branch locus. If $\Pi(v_2) \neq \Pi(v_3)$, then $\Pi(v_1)$ intersects the bridge between $\Pi(v_1)$ and $\Pi(v_i)$ $(i = 2, 3)$ at a point which is contained in the branch locus, a contradiction to the fact that v_1 is a nonsingular rational curves. We may also assume that $\Pi(v_2) = \Pi(v_3)$, it is not a component of the branch locus and its multiplicity is equal to two. Since the multiplicity of double covering of $\widetilde{\Gamma}_0$ is equal to two, there exists the bridge between $\Pi(v_2)$ and $\widetilde{\Gamma}_0$ which dose not contain $\Pi(v_1)$ as a component. However, $\widetilde{\Gamma}_0$ is a component of branch locus, a contradiction to the configuration of the special fiber. Assume that $\Pi(v_1)$ is a component of the branch locus. Then the multiplicity of $\Pi(v_1)$ is equal to two and $\Pi(v_2) \neq \Pi(v_3)$. Since the multiplicities of $\Pi(v_4)$ and the bridge between $\Pi(v_4)$ and $\Pi(v_1)$ is greater than one and even, $\Pi(v_4)$ and the bridge between $\Pi(v_4)$ and $\Pi(v_1)$ can be contracted to a point on $\Pi(v_1)$ by inverse of Horikawa's canonical resolution. Since the multiplicity of $\Pi(v_1)$ is equal to two, there is two divisor E, E' whose multiplicity are equal to one and intersect $\Pi(v_1)$ transversally. If E (or E') is not a component of the branch locus, it is a contradiction to the fact that Γ_0 is the branch locus of ψ_r. Then we may assume E and E' are the component of branch locus. However, in view of the configuration of the special fibre, there is no components whose multiplicity is three, a contradiction.

Case N: II(2,5). If there exists a hyperelliptic family S whose topological monodromy is N: II(2,5), then the figure of the special fiber is as in Figure 2. The lines mean nonsingular rational curves and the numbers beside them are their multiplicities. Since there exists a loop containing C_2, $\Pi(C_2)$ is not a component of the branch locus. Considering its stable model, the number of rational components between C_1 and C_3 are the same as the number of rational components between C_1 and C_4. If $\Pi(C_1)$ is a component of the branch locus, $\Pi(C_3)$ is not equal to $\Pi(C_4)$. Considering the inverse of the canonical resolution, we can contract $\Pi(C_3)$ and $\Pi(C_4)$ to two distinct points P_1 and P_2 on $\Pi(C_1)$. Moreover, in view of the figure of the special fiber, we may assume that the local equation for the branch locus at P_i $(i = 1, 2)$ is given by $t(x^2 - t^p)$ $(p \in \mathbf{N})$. Let P_3 be a point on $\Pi(C_1)$ at which the other components of Θ are contracted and let $tF(x, t)$ be a local equation for the branch locus at P_3. We may assume that S_0 is defined by the equation

$$y^2 = tF(x,t)\{(x-1)^2 - t^p\}\{(x+1)^2 - t^p\}.$$

The semistable model of S_0 is given by the resolution of singularities of the surface which is defined by the equation $y^2 = F(x, t^2)\{(x-1)^2 - t^{2p}\}\{(x+1)^2 - t^{2p}\}$. Assume that the special fiber of the stable model of S_0 is (N). By an easy calculation, we see that the special fiber of the stable model of the surface defined by the equation $y^2 = F(x, t^2)\{(x-1)^2 - t^{2p}\}\{(x+1)^2 - t^{2p'}\}$ $(p \neq p')$ is also (N). However, the stable model of the surface defined by $y^2 = tF(x,t)\{(x-1)^2 - t^2\}\{(x+1)^2 - t^{p'}\}$ is not (N), because the number of rational components between C_1 and C_3 is different from that of C_1 and C_4, a contradiction. Hence we may assume that $\Pi(C_1)$ is not a component of the branch locus and its multiplicity is two. If $\Pi(C_3)$ is not equal to $\Pi(C_4)$, then we have a contradiction by Lemma 3.1. We may also assume that $\Pi(C_3)$ is equal to $\Pi(C_4)$ and its multiplicity is two. In view of the special fiber of S and Lemma 3.1, Θ has to be as in Figure 3. In Figure 3, the dotted lines mean the components of Θ which are not components of the branch locus and the solid lines mean the components of the branch locus. The waves mean the branch locus which are not components of Θ. The monodromy of the family of the double covering of the branch locus as in Figure 3 is F_{00}: II(1,6).

Case E_{11}: II(0,1), $V_1 = V_2 =$ (iv3) and the screw number at e_1 is special. In this case, the special fiber is as in Figure 4. The screw number "special" in this case means that the number of components of nonsingular rational curves between v_1 and v_2 is even.

If $\Pi(v_1)$ is equal to $\Pi(v_2)$, none of $\Pi(v_i)$ $(1 \leq i \leq 8)$ are the components of the branch locus, contradictory to Lemma 3.1. Assume that $\Pi(v_1)$ is not a component of the branch locus. Since $\Pi(v_5)$ is not a component of the branch locus, the bridge between $\Pi(v_1)$ and $\Pi(v_5)$ intersects $\Pi(v_1)$ at a point which is contained in the branch locus. By the same argument, the bridge between $\Pi(v_1)$ and $\Pi(v_7)$ intersects $\Pi(v_1)$ at a point which is contained in the branch locus. Since v_1 is a nonsingular rational curve, the bridge between $\Pi(v_1)$ and $\Pi(v_3)$ intersects $\Pi(v_1)$ at a point which is not contained in the branch locus, a contradiction to the configuration of the special fiber as in Figure 4. Thus, $\Pi(v_1)$ is a component of the branch locus. By the same argument, $\Pi(v_2)$ is also a component of the branch locus. If $\Pi(v_3)$ is the component of the branch locus, the configuration of the components of Θ is as in Figure 5. Since the double covering of the bridge between $\Pi(v_1)$ and $\Pi(v_3)$ contracts to a point at which v_1 intersects v_3, the multiplicities of the components of the bridge between $\Pi(v_1)$ and $\Pi(v_3)$ are greater than four. Moreover, since the multiplicity of $\Pi(v_3)$ is one and Θ has to be obtained by Horikawa's canonical resolution, there exists a composite of blowing-downs τ' of (-1)-curves such that the divisor D as in Figure 5 has to be contracted to a point on $\tau'(\Pi(v_3))$. It means that $\Pi(v_1)$ cannot be contracted before all of the components of the bridge between $\Pi(v_1)$ and $\Pi(v_3)$ by the inverse of Horikawa's canonical resolution. We thus have a contradiction to the assumption that the double covering of the bridge between $\Pi(v_1)$ and $\Pi(v_3)$ contracts to a point. By the same argument, $\Pi(v_4)$ is not the component of the branch locus. Assume that there is the bridge $D_{v_1v_3}$ between $\Pi(v_1)$ and $\Pi(v_3)$, i.e., $\Pi(v_1)$ and $\Pi(v_3)$ do not intersects at a point. Note that in view of the configuration of the special fibre, the multiplicities of the components of $D_{v_1v_3}$ are greater than four, i.e, the multiplicities of the components of $D_{v_1v_3}$ are greater than that of $\Pi(v_1)$ and $\Pi(v_3)$ and $D_{v_1v_3}$ dose not intersect the strict transform of $\widetilde{B}_0$ because of the configuration of the double covering of the bridge. Considering the inverse of Horikawa's canonical resolution, we see that there exists W_j such that $(\tau_j \circ \cdots\cdots \tau_r)(D_{v_1v_3})$ is a point, $(\tau_j \circ \cdots \circ \tau_r)(\Pi(v_1))$ intersects $(\tau_j \circ \cdots \circ \tau_r)(\Pi(v_3))$ at a point P and the strict transform of $\widetilde{B}_0$ dose not contain P, a contradiction to the process of Horikawa's canonical resolution. Thus, $\Pi(v_1)$ intersects $\Pi(v_3)$ at a point. By the same argument, $\Pi(v_1)$ intersects $\Pi(v_l)$ $(l = 5, 7)$ at a point, respectively and $\Pi(v_2)$ intersects $\Pi(v_{l'})$ $(l' = 4, 6, 8)$ at a point, respectively. Then the configuration of Θ is as in Figure 6, (a). Let $\sum_{i_j=1}^{j=k} a_{i_j} E_{i_j}$ be the bridge between $\Pi(v_3)$ and $\Pi(v_4)$ such that $\Pi(v_3)E_{i_1} = 1$, $\Pi(v_4)E_{i_k} = 1$, $E_{i_j}E_{i_{j+1}} = 1$ $(1 \leq j \leq k-1)$ and $E_{i_j}E_{i_{j'}} = 0$ $(|j - j'| \geq 2)$. Consider the inverse of Horikawa's canonical resolution such that all but $\Pi(v_8)$ are contracted to a point. First, we contract $\Pi(v_5)$ and $\Pi(v_6)$, then contract $\Pi(v_1)$ and $\Pi(v_2)$. Then the configuration of the image of Θ by above contractions is as in Figure 6, (b). We use the same name of the components of Θ after contractions.

Assume a_{i_1} not to be one. Since we cannot contract $\Pi(v_3)$ in the next step, there exists j' such that $a_{i_{j'}} = 1$ and after sum steps of blowing-downs, $E_{i_{j'}}$ intersects $\Pi(v_3)$ at a point. However, $\sum_{i_j=1}^{j=k} a_{i_j} E_{i_j}$ dose not intersects the strict transform of $\widetilde{B}_0$, a contradiction to the process of Horikawa's canonical resolution. Thus, a_{i_1} is equal to one. By the same argument we see that a_{i_k} is equal to one. If a_{i_2} is greater than two, a_{i_3} is greater than two. Then we have $a_{i_j} < a_{j'}$ for all $j < j'$, a contradiction to $a_{i_k} = 1$. Thus we see that $a_{i_2} = 2$. Applying above consideration to a_{i_2} in place of $\Pi(v_3)$, we obtain that $a_{i_j} = 1$ when j is odd and $a_{i_j} = 2$ when j is even. Then the number of components of nonsingular rational curves between v_1 and v_2 is odd.

Other cases are proved by similar arguments. q.e.d.

3.2 Periodic case

Let $S_d \longrightarrow \Delta$ be a degeneration of curves of genus g obtained by a base change of degree d of $S \longrightarrow \Delta$. Taking F as a representative of the monodromy of S (indeed, F is a pseudo-periodic map and its conjugacy class $[F]$ in the mapping class group is the monodromy of S), the monodromy of S_d is $[F^d]$.

In Lemma 1.4 of [AI], we classified the conjugacy class of periodic maps of genus g $(1 \leq g \leq 3)$. The data of the conjugacy class of a periodic map $[f]$ consist of two invariants: the period and the total valency. The period n means that $(f)^n$ is isotopic to the identity. If we take a suitable representative F of the isotopy class of f, the data of period n and total valency $n_1/m_1 + n_2/m_2 \cdots + n_k/m_k$ has the following local data: there exist n/m_i points $P_1, P_2, \cdots, P_{n/m_i}$ such that $F^{n/m_i}(P_j) = P_j$ $(j = 1, 2, \cdots, n/m_i)$ and F^{n/m_i} is isotopic to the rotation of angle $2\pi \times \delta_i/m_i$ near P_j, where δ_i is the integer which satisfies $\delta_i n_i \equiv 1 \pmod{m_i}$.

For instance, (iii)-(1) in [AI, Lemma 1.4], $n = 14$; 1/14+3/7+1/2 means that there exist points P_k $(1 \leq k \leq 10)$ which satisfy $F(P_1) = P_1$, $F^2(P_i) = P_i$ $(i = 2, 3)$, $F^7(P_j) = P_j$ $(4 \leq j \leq 10)$, respectively. Moreover, F is isotopic to the rotation of angle $2\pi/14$ near P_1, F^2 isotopic to the rotation of angle $2\pi \times 5/7$ near P_2 and P_3, and F^7 is isotopic to the rotation of angle $2\pi \times 1/2$ near P_j $(4 \leq j \leq 10)$. By an easy calculation, we see that F^7 is a periodic map of $n = 2$; 1/2+1/2+1/2+1/2+1/2+1/2+1/2+1/2. Let S be a family of genus three whose monodromy is a periodic map F, $n = 14$; 11/14+7/5+1/2. Taking a base change of degree seven, we obtain a family $S_d \longrightarrow \Delta$ whose monodromy is a periodic map, $n = 2$; 1/2+1/2+1/2+1/2+1/2+1/2+1/2+1/2 by the above observations. Repeating this calculations for all periodic maps of genus three, we obtain the following lemma:

Lemma 3.5 *By taking a base change of suitable degree, all of periodic maps of genus three are obtained from*
(i1) $n = 14,\ 11/14 + 3/7 + 1/2,$
(i7) $n = 12,\ 11/12 + 7/12 + 1/2,$
(i9) $n = 12,\ 11/12 + 3/4 + 2/3,$
(i13) $n = 9,\ 8/9 + 4/9 + 2/3,$
(i20) $n = 8,\ 1/8 + 5/8 + 1/4,$
(i22) $n = 8,\ 3/8 + 3/8 + 3/4,$
(i28) $n = 7,\ 1/7 + 2/7 + 4/7,$
(i44) $n = 4,\ g' = 1,\ \ 1/2 + 1/2,$
(i47) $n = 2,\ g' = 2$ *and* $\Pi\colon \Sigma_g \longrightarrow \Sigma_{g'}$ *is an unramified covering.*

Proof Let $h_1\colon \Sigma_3 \longrightarrow \Sigma_3$ be a homeomorphism whose conjugacy class in the mapping class group has data $n = 14,\ 11/14 + 3/7 + 1/2$. Using the same symbols in §2.2, we write $m_1 =$ (i1), for short. We set $m_7 :=$ (i7), $m_9 :=$ (i9), $m_9 :=$ (i9), $m_{13} :=$ (i13), $m_{20} :=$ (i20), $m_{22} :=$ (i22), $m_{28} :=$ (i28), $m_{44} :=$ (i44), $m_{47} :=$ (i47). Then, by elementary calculations, we obtain the following equations:

$(m_1)^2 =$ (i31), $(m_1)^3 =$ (i3), $(m_1)^4 =$ (i25), $(m_1)^5 =$ (i6), $(m_1)^6 =$ (i29),
$(m_1)^7 =$ (i43), $(m_1)^8 =$ (i30), $(m_1)^9 =$ (i5), $(m_1)^{10} =$ (i26), $(m_1)^{11} =$ (i4),
$(m_1)^{12} =$ (i32), $(m_1)^{13} =$ (i2), $(m_7)^2 =$ (i33), $(m_7)^3 =$ (i39), $(m_7)^4 =$ (i45),
$(m_7)^7 =$ (i8), $(m_7)^9 =$ (i40), $(m_9)^2 =$ (i34), $(m_9)^3 =$ (i36), $(m_9)^4 =$ (i41),
$(m_9)^5 =$ (i11), $(m_9)^6 =$ (i46), $(m_9)^7 =$ (i12), $(m_9)^8 =$ (i42), $(m_9)^{10} =$ (i35),
$(m_9)^{11} =$ (i10), $(m_{13})^2 =$ (i16), $(m_{13})^4 =$ (i18), $(m_{13})^5 =$ (i17), $(m_{13})^7 =$ (i15),
$(m_{13})^8 =$ (i14), $(m_{20})^3 =$ (i19), $(m_{20})^2 =$ (i37), $(m_{22})^3 =$ (i24), $(m_{20})^6 =$ (i138)
$(m_{22})^5 =$ (i23), $(m_{22})^7 =$ (i21), $(m_{28})^3 =$ (i27).

q.e.d.

Theorem 3.6 *There exist hyperelliptic families whose topological monodromies are the following:*
(i1) $n = 14,\ 11/14 + 3/7 + 1/2,$
(i7) $n = 12,\ 11/12 + 7/12 + 1/2,$
(i22) $n = 8,\ 3/8 + 3/8 + 3/4,$
(i44) $n = 4,\ g' = 1,\ \ 1/2 + 1/2,$
(i47) $n = 2,\ g' = 2$ *and* $\Pi\colon \Sigma_g \longrightarrow \Sigma_{g'}$ *is an unramified covering.*

For any hyperelliptic families, the topological monodromy is obtained by a base change of suitable degree from one of the above monodromies.

Proof We prove the existence of the families by giving examples of the equations for the families in Section 4. By Lemma 3.5, it suffices to prove that there exist no hyperelliptic families whose monodromies are the following: (i) $n = 7$; 1/7+2/7+4/7, (ii) $n = 6$; 1/6+2/3+2/3+1/2, (iii) $n = 4$; 1/4+1/4+1/4+1/4, (iv) $n = 3$; 1/3+1/3+1/3+1/3+2/3.

Case (i) Assume that there exists a hyperelliptic family S whose monodromy is (i). Let C_0, C_1, C_2, C_3 be the components of the special fiber of S whose multiplicities are 7, 1, 2 and 4, respectively and C_0 intersects C_1, C_2 and C_3. Since $\Pi(C_i)$ are all distinct, $\Pi(C_0)$ intersects distinct three subdivisors of $(\tau_1 \circ \cdots \circ \tau_r)^*(\pi^{-1}(0))$, a contradiction to Lemma 3.1, because the greatest common divisor of their multiplicities is one.

Case (ii) and Case (iv) are similar to Case (i).

Case (iii) Assume that there exists a hyperelliptic family S whose monodromy is (iii). Let C_0, C_i $(i = 1, 2, 3, 4)$ be the components of the special fiber of S whose multiplicities are 4 and 1, respectively. If $\Pi(C_0)$ is a component of the branch locus of ϕ_r, we have a contradiction to Lemma 3.1. Thus we may assume that $\Pi(C_0)$ is not a component of the branch locus, $\Pi(C_1) = \Pi(C_2)$ and $\Pi(C_3) = \Pi(C_4)$. We may also assume that $\Pi(C_0)$ intersects $\Pi(C_1)$ and $\Pi(C_3)$ at one point, respectively, because the bridges between $\Pi(C_0)$ and $\Pi(C_j)$ $(j = 1, 3)$ do not intersect the branch locus of ϕ_r. In this case, there exist no components of $(\tau_1 \circ \cdots \circ \tau_r)^*(\pi^{-1}(0))$ whose self-intersection number is -1, a contradiction. q.e.d.

4 Example of the equations

In this section, for each conjugacy class of pseudo-periodic maps $[F]$ which can be realized as the monodromy of a certain hyperelliptic family, we give an examples of a hyperelliptic family (ϕ, S, Δ) whose monodromy is $[F]$. More precisely, we give an equation of a double covering S_0 of $\mathbf{P}^1 \times \Delta$, and S is the normally minimal model of S_0. Indices which appear in the table of symbols and equations are positive integers unless we mention their range. Let α, α_i $(1 \leq i \leq 4)$ be real numbers which are not integers and mutually distinct. Let (x, t) be a local coordinate of $\mathbf{P}^1 \times \Delta$. For example, we give an equation for S_0 whose topological monodromy is (A_3) as follows:

$$(A_3) \quad y^2 = x(x-1)(x-2)(x-3)(x-4)(x-5)(x-6)(x-7).$$

We introduce some symbols for simplicity.
$F_3(x,t,k) := x^3 - t^k$,
$F_{12}(x,t,K_1,K_2) := (x - t^{K_1})(x^2 - t^{K_2})$,
$\widetilde{F}_3(x,t,K,L) := (x^2-t)^3 - t^K x^L$,
$\widetilde{F}_4(x,t,K,L) := (x^2-t)^4 - t^K x^L$,
$\widetilde{F}_{12}(x,t,K_1,K_2,L_1,L_2) := \{(x^2-t) - t^{K_1}x^{L_1}\}\{(x^2-t)^2 - t^{K_2}x^{L_2}\}$,
$\widetilde{F}_{13}(x,t,K_1,L_1,K_2,L_2) := \{(x^2-t) - t^{K_1}x^{L_1}\}\{(x^2-t)^3 - t^{K_2}x^{L_2}\}$,
$\widetilde{F}_{22}(x,t,K,L) := \{(x^2-t)^2 - t^K x^L\}\{(x^2-t)^2 + t^K x^L\}$.

Let c be a positive integer. We fix a pair of integers (K',L') which satisfies $2K'+L'-6=c$, $K'>0$ and $0 \le L' < 5$. We set $\widetilde{F}_3^c := \widetilde{F}_3(x,t,K',L')$. Similarly, fixing a pair of (K',L') which satisfies $2K'+L'-8=c$, $K'>0$ and $0 \le L' < 8$, we set $\widetilde{F}_4^c := \widetilde{F}_4(x,t,K',L')$. Fixing a pair of (K',L'), we set $\widetilde{F}_{22}^c := \widetilde{F}_{22}(x,t,K',L')$. Let c_1 and c_2 be positive integers. We fix two pair of integers (K_1',L_1') and (K_2',L_2') which satisfy $2K_1'+L_1'-2=c_1$, $2K_2'+L_2'-4=c_2$, $K_1'>1$, $0 \le L_1' \le 1$, $K_2'>1$ and $0 \le L_2' \le 3$. We set $\widetilde{F}_{12}^{c_1,c_2} := F_{12}(x,t,K_1',L_1',K_2',L_2')$. Fixing two pair of integers (K_1',L_1') and (K_2',L_2') which satisfy $2K_1'+L_1'-2=c_1$ and $2K_2'+L_2'-6=c_2$, we set $\widetilde{F}_{13}^{c_1c_2} := \widetilde{F}_{13}(x,t,K_1',L_1',K_2',L_2')$
We define the following symbols using the above symbols.

$f_1(x,t,k,l) := x^3 - \alpha_1 t^{6(k-1)}$. $f_2(x,t,k,l) := F_3(x,t,6k+3l-1)$.
$f_3(x,t,k,l) := F_3(x,t,6k+3l-5)$. $f_4(x,t,k,l) := F_3(x,t,6k+3l-2)$.
$f_5(x,t,k,l) := F_3(x,t,6k+3l-4)$. $f_6(x,t,k,l) = F_{12}(x,t,2k+l,4k+2l-1)$.
$f_7(x,t,k,l) := F_{12}(x,t,2k+l,4k+2l-3)$. $f_8(x,t,k,l) := F_3(x,t,6k+3l-3)$.
$g_1(x,t,k) := x^5 - \alpha_2 t^{10(k-1)}$. $g_2(x,t,k) := x^5 - t^{10k-7}$.
$g_3(x,t,k) := x^5 - t^{10k-3}$. $g_4(x,t,k) := x^5 - t^{10k-1}$.
$g_5(x,t,k) := x^5 - t^{10k-9}$. $g_6(x,t,k) := x(x^4 - t^{8k-1})$.
$g_7(x,t,k) := x(x^4 - t^{8k-3})$. $g_8(x,t,k) := x(x^4 - t^{8k-7})$.
$g_9(x,t,k) := x(x^4 - t^{8k-5})$. $g_{13}(x,t,k) := x^5 - t^{10k-4}$.
$g_{15}(x,t,k) := x^5 - t^{10k-6}$. $g_{17}(x,t,k) := x^5 - t^{10k-2}$.
$g_{19}(x,t,k) := x^5 - t^{10k-8}$. $g_{20}(x,t,k) := x(x^2 - t^{4k-1})(x^2 + t^{4k-1})$.
$g_{21}(x,t,k) := x(x^2 - t^{4k-3})(x^2 + t^{4k-3})$. $g_{24}(x,t,k) := (x^5 - t^{10k-5})$.
$h_1(x,t,k,l) := F_{12}(x,t,2l,k+4l)$. $h_1(x,t,k,l) := F_{12}(x,t,2l-1,k+4l-2)$.
$\sigma_1(x,t,k,l) := (x^3 - t^{6l})(x^2 - t^{k+4l})$. $\sigma_2(x,t,k,l) := (x^3 - t^{6k-2})(x^2 - t^{4k+l})$.
$\sigma_3(x,t,k,l) := (x^3 - t^{6k-4})(x^2 - t^{4k+l-1})$. $\sigma_5(x,t,k,l) := (x^3 - t^{6k-1})(x^2 - t^{4k+l})$.
$\sigma_6(x,t,k,l) := (x^3 - t^{6k-5})(x^2 - t^{4k+l-2})$. $\sigma_9(x,t,k,l) := (x^3 - t^{6k-3})(x^2 - t^{4k+l-2})$.
$\tau_1(x,t,k_1,k_2) := F_{12}(x,t,2,k_1+4)\{(x-t^2)^2 - t^{k_2+2}\}$.

$\tau_2(x,t,k_1,k_2) := (x-2t)(x^2-t^{k_1+2})\{(x-t)^2-t^{k_2+2}\}$.
$\tau_4(x,t,k_1,k_2) := (x-t^2)\{(x^2-t^3)^2-t^{k_1}x^{k_2}\}$, $(2k_1+k_2-6\geq 1)$.
$\tau_5(x,t,k_1,k_2) := x\{(x^2-t)^2-t^{k_1}x^{k_2}\}$ $(2k_1+k_2-4\geq 1)$.
$\theta_1 := \widetilde{F}_3^{6k+3}$. $\theta'_1(x,t,k) := \widetilde{F}_3^{6k+6}$. $\theta_2(x,t,k) := \widetilde{F}_3^{6k+2}$.
$\theta_3(x,t,k) := \widetilde{F}_3^{6k-2}$. $\theta'_3(x,t,k) := \widetilde{F}_3^{6k+1}$. $\theta_4(x,t,k) := \widetilde{F}_3^{6k+1}$.
$\theta'_4(x,t,k) := \widetilde{F}_3^{6k+4}$. $\theta_5(x,t,k) := \widetilde{F}_3^{6k-1}$. $\theta'_5(x,t,k) := \widetilde{F}_3^{6k+2}$.
$\theta_6(x,t,k) := \widetilde{F}_{12}^{2k+1,\ 4k+1}$. $\theta'_6(x,t,k) := \widetilde{F}_{12}^{2k+2,\ 4k+3}$. $\theta_7(x,t,k) := \widetilde{F}_{12}^{2k+1,\ 4k-1}$.
$\theta'_7(x,t,k) := \widetilde{F}_{12}^{2k+1,\ 4k+1}$. $\theta_8(x,t,k) := \widetilde{F}_3^{6k}$. $\theta'_8(x,t,k) := \widetilde{F}_3^{6k+3}$.
$\omega_1(x,t,k) := x^4-t^{4(k-1)}$. $\omega'_1(x,t,k) = \widetilde{F}_4^{4k}$. $\omega_2(x,t,k) := x^4-t^{4k-1}$.
$\omega'_2(x,t,k) := \widetilde{F}_4^{4k+6}$. $\omega_3(x,t,k) := x^4-t^{4k-3}$. $\omega'_3(x,t,k) := \widetilde{F}_4^{4k+1}$.
$\omega_4(x,t,k) := (x-t^k)(x^3-t^{3k-1})$. $\omega'_4(x,t,k) := \widetilde{F}_4^{3k+2}$.
$\omega_5(x,t,k) := (x-t^k)(x^3-t^{3k-2})$. $\omega'_5(x,t,k) := \widetilde{F}_{22}^{2k+3}$.
$\omega_6(x,t,k) := (x^2-t^{2k+1})(x^2+t^{2k+1})$. $\omega'_6(x,t,k) := \widetilde{F}_{13}^{k+1,3k+2}$.
$\Gamma_1(x,t,k) = (x-t^k)(x^3-t^{3k-2})$. $\Gamma_2(x,t,k) := (x-t^k)(x^3-t^{3k-1})$.
$\Gamma_3(x,t,k) := x^4-t^{4k-3}$. $\Gamma_4(x,t,k) := x^4-t^{4k-1}$.
$\Gamma_5(x,t,k) := x^4-\alpha t^{4k-4}$. $\Gamma_6(x,t,k) := x^4-t^{4k-2}$.
$\rho_1(x,t,k,l) := (x^2-\alpha t^{2(l-1)})(x^2-t^{k+2(l-1)})$. $\rho'_1(x,t,k,l) := \rho_1(x,t,k,l+1/2)$
$\rho_2(x,t,k,l) := (x^2-t^{2l-1})(x^2-t^{k+2l-1})$. $\rho'_2(x,t,k,l) := \rho_2(x,t,k,l+1/2)$.
$\eta_1(x,t,k,l) = \tilde{F}_{13}^{l,6k+3l-3}$. $\eta_2(x,t,k,l) = \tilde{F}_{13}^{l,6k+3l-4}$. $\eta_3(x,t,k,l) = \tilde{F}_{13}^{l,6k+3l-8}$.
$\eta_4(x,t,k,l) = \tilde{F}_{13}^{l,6k+3l-5}$. $\eta_5(x,t,k,l) = \tilde{F}_{13}^{l,6k+3l-7}$.
$\eta_6(x,t,k,l) = \widetilde{F}_{12}^{l,4k-5}\{(x^2-t)-t^{k_1}x^{l_1}\}$, $(2k_1+l_1=l+2)$.
$\eta_7(x,t,k,l) = \widetilde{F}_{12}^{l,4k-7}\{(x^2-t)-t^{k_1}x^{l_1}\}$, $(2k_1+l_1=l+2)$. $\eta_8(x,t,k,l) = \tilde{F}_{13}^{l,6(k-1)+3l}$.

First we give examples of semistable curves. Next we give examples of hyperelliptic families whose topological monodromies are periodic. According to Lemma 3.5, it is sufficient to give only five examples listed in Theorem 3.6. At the end of this section, we give examples of hyperelliptic families whose topological monodromies are neither periodic nor semistable. We have to give two or three equations for the same symbol of the topological monodromies classified in [AI] because of the difference of their screw numbers.

The cases of semistable curves

(A_3) $y^2 = x(x-1)(x-2)(x-3)(x-4)(x-5)(x-6)(x-7)$.
(A_2) $y^2 = (x^2-t^k)(x-1)(x-2)(x-3)(x-4)(x-5)(x-6)$.
(A_1) $y^2 = (x^2-t^{k_1})\{(x-1)^2-t^{k_2}\}(x-2)(x-3)(x-4)(x-5)$.
(A_0) $y^2 = (x^2-t^{k_1})\{(x-1)^2-t^{k_2}\}\{(x-2)^2-t^{k_3}\}(x-3)(x-4)$.

$(B_{21})\quad y^2=(x^3-t^{6k})(x-1)(x-2)(x-3)(x-4)(x-5).$

$(B_{20})\quad y^2=(x-t^{2k})(x^2-t^{4k+l})(x-1)(x-2)(x-3)(x-4)(x-5).$

$(B_{11})\quad y^2=(x^3-t^{6k_1})\{(x-1)^2+t^{k_2}\}(x-2)(x-3)(x-4).$

$(B_{10})\quad y^2=(x-t^{2k_1})(x^2-t^{4k_1+l})\{(x-1)^2+t^{k_2}\}(x-2)(x-3)(x-4).$

$(B_{01})\quad y^2=(x^3-t^{6k_1})\{(x-1)^2+t^{k_2}\}\{(x-2)^2+t^{k_3}\}(x-3).$

$(B_{00})\quad y^2=(x-t^{2k_1})(x^2-t^{4k_1+l}))\{(x-1)^2+t^{k_2}\}\{(x-2)^2+t^{k_3}\}(x-4).$

$(C_{111})\quad y^2=(x^3-t^{6k_1})\{(x-1)^3-t^{6k_2}\}(x-3)(x-4).$

$(C_{101})\quad y^2=(x^3-t^{6k_1})\{(x-1)^2-t^{l}\}\{(x-2)^3-t^{6k_2}\}.$

$(C_{011})\quad y^2=(x-t^{2k_1})(x^2-t^{4k_1+l})\{(x-1)^3-t^{6k_2}\}(x-3)(x-4).$

$(C_{010})\quad y^2=(x+t^{2k_1})(x^2-t^{4k_1+l_1})(x-2+t^{2k_2})\{(x-2)^2-t^{2(2k_1+l_2)}\}(x-3)(x-4).$

$(C_{000})\quad y^2=(x+t^{2k_1})(x^2-t^{4k_1+l_1})\{(x-2)^2-t^{l_2}\}(x-3+t^{2k_2})\{(x-3)^2-t^{4k_1+l_3}\}.$

$(E_{11})\quad y^2=(x^4-t^{4k})(x-1)(x-2)(x-3)(x-4).$

$(E_{01})\quad y^2=(x^2-t^{2k})(x^2-\alpha t^{2k+2l_1+l_2})(x-1)(x-2)(x-3)(x-4).$

$(E_{00})\quad y^2=(x^2-t^{2k})(x^2-t^{2k+2l_1+l_2})\{(x-1)^2-t^{l_3}\}(x-3)(x-4).$

$(F_{11})\quad y^2=(x-\alpha t^{k_2})(x^3-t^{3k_2+3(2l+1)k_3})\{(x-1)^3-t^{6k_1}\}(x-2).$

$(F_{01})\quad y^2=(x-\alpha t^{k_2})(x^3-t^{3k_2+3(2l+1)k_3})(x-1+t^{2k_1})\{(x-1)^2-t^{4k_1+l_1}\}(x-2).$

$(F_{00})\quad y^2=(x-1+t^{2k_1})\{(x-1)^2-t^{4k_1+l_1}\}(x-\alpha t^{k_2})(x-t^{k_2+2k_3})(x^2-t^{2k_2+4k_3+l_2}).$

$(G_{11})\quad y^2=(x-t^{k_1})(x^3-t^{3k_1+6k_2})(x-1)(x-2)(x-3)(x-4).$

$(G_{10})\quad y^2=(x-\alpha t^{k_1})(x-t^{k_1+2k_2})(x^2-t^{2k_1+2(2k_2+l)})(x-1)(x-2)(x-3)(x-4).$

$(G_{01})\quad y^2=(x-t^{k_1})(x^3-t^{3k_1+6k_2})\{(x-1)^2-t^{l_2}\}(x-2)(x-3).$

$(G_{00})\quad y^2=(x-\alpha t^{k_1})(x-t^{k_1+2k_2})(x^2-t^{2k_1+4k_2+l_1})\{(x-1)^2-t^{l_2}\}(x-2)(x-3).$

$(J_1)\quad y^2=(x^4-t^{4k_1})\{(x-1)^2-t^{k_2}\}\{(x-2)^2-t^{k_3}\}.$

$(J_0)\quad y^2=(x^2-t^{2k_1})(x^2-t^{2k_1+l_1})\{(x-1)^2-t^{k_2}\}\{(x-2)^2-t^{k_3}\}.$

$(K_1)\quad y^2=(x-t^{k_3})(x^3-t^{3k_3+3(2l+1)k_4})\{(x-1)^2-t^{k_1}\}\{(x-2)^2-t^{k_2}\}.$

$(K_0)\quad y^2=(x-\alpha t^{k_3})(x-t^{k_3+2k_4})(x^2-t^{2k_3+4k_4+l})\{(x-1)^2-t^{k_1}\}\{(x-2)^2-t^{k_2}\}.$

$(L)\quad y^2=(x^2-t^{k_1})\{(x-1)^2-t^{k_2}\}\{(x-2)^2-t^{k_3}\}\{(x-3)^2-t^{k_4}\}.$

$(N)\quad y^2=\{(x-t^{k_1})^2-t^{2k_1+k_2+1}\}\{x^2-t^{2k_1+k_3+1}\}\{(x-1)^2-t^{k_4}\}\{(x-2)^2-t^{k_5}\}.$

The periodic cases

(i1) $\quad y^2=(x^7-t^{11})(x-1).$

(i7) $\quad y^2=x(x^6-t^{11})(x-1).$

(i22) $\quad y^2=x^8+t^3.$

(i44) $\quad y^2=t\{(x^4-t)(x^4+t)\}.$

(i47) $\quad y^2=t\{(x^2-\alpha_1 t)(x^2-\alpha_2 t)(x^2-\alpha_3 t)(x^2-\alpha_4 t)\}.$

Next, we give examples of hyperelliptic families whose stable models are neither smooth curves nor semistable curves. In this case, the equations are classified by the types of the stable models (cf. Theorem 3.4).

The cases where the stable model is A_2

(iii4) $y^2 = t(x^6 - t^5)(x^2 - t)$, $y^2 = (x^6 - t^5)(x - 1)(x - 2)$, $y^2 = (x^6 - t^5)\{(x - 1)^2 - t^k\}$.

(iii5) $y^2 = (x^6 - t)(x - 1)(x - 2)$, $y^2 = (x^6 - t)\{(x - 1)^2 - t^k\}$.

(iii6) $y^2 = x(x^5 - t^4)(x - 1)(x - 2)$, $y^2 = x(x^5 - t^4)\{(x - 1)^2 - t^k\}$.

(iii8) $y^2 = x(x^5 - t)(x - 1)(x - 2)$, $y^2 = x(x^5 - t)\{(x - 1)^2 - t^k\}$.

(iii10) $y^2 = x(x^5 - t^2)(x - 1)(x - 2)$, $y^2 = x(x^5 - t^2)\{(x - 1)^2 - t^k\}$.

(iii12) $y^2 = tx(x^2-t)(x^5-t^3)$, $y^2 = x(x^5-t^3)(x-1)(x-2)$, $y^2 = x(x^5-t^3)\{(x-1)^2-t^k\}$.

(iii15) $y^2 = t(x^3 + t^2)(x^3 - t^2)(x^2 - t)$, $y^2 = (x^3 - t^2)(x^3 + t^2)(x - 1)(x - 2)$,
$y^2 = (x^3 - t^2)(x^3 + t^2)\{(x - 1)^2 - t^k\}$.

(iii17) $y^2 = (x^3 - t)(x^3 + t)(x - 1)(x - 2)$, $y^2 = (x^3 - t)(x^3 + t)\{(x - 1)^2 - t^k\}$.

(iii19) $y^2 = (x^2-t)(x^2+t)(x^2-2t)(x-1)(x-2)$, $y^2 = (x^2-t)(x^2+t)(x^2-2t)\{(x-1)^2-t^k\}$.

(iii20) $y^2 = x(x^5 - t^2)(x - 1)(x - 2)$, $y^2 = x(x^5 - t^2)\{(x - 1)^2 - t^k\}$.

(iii21) $y^2 = t(x^5 - t^2)(x^2 - t)(x - 1)$, $y^2 = x(x^5 - t^3)(x - 1)(x - 2)$,
$y^2 = x(x^5 - t^3)\{(x - 1)^2 - t^k\}$.

(iii22) $y^2 = tx(x^5 - t)(x - 1)(x - 2)$, $y^2 = tx(x^5 - t)\{(x - 1)^2 - t^k\}$.

(iii23) $y^2 = (x^2 - t)(x^5 - t)(x - 1)$, $y^2 = tx(x^5 - t^4)(x - 1)(x - 2)$,
$y^2 = tx(x^5 - t^4)\{(x - 1)^2 - t^k\}$.

(iii24) $y^2 = t(x^6 - t)(x - 1)(x - 2)$, $y^2 = t(x^6 - t)\{(x - 1)^2 - t^k\}$.

(iii25) $y^2 = (x^2 - t)(x^6 - t)$, $y^2 = t(x^6 - t^5)(x - 1)(x - 2)$, $y^2 = t(x^6 - t^5)\{(x - 1)^2 - t^k\}$.

(iii26) $y^2 = t(x^3 - t)(x^3 + t)(x - 1)(x - 2)$, $y^2 = t(x^3 - t)(x^3 + t)\{(x - 1)^2 - t^k\}$.

(iii27) $y^2 = t(x^3 - t)(x^3 + t)(x^2 - t)$, $y^2 = t(x^3 - t^2)(x^3 + t^2)(x - 1)(x - 2)$,
$y^2 = t(x^3 - t^2)(x^3 + t^2)\{(x - 1)^2 - t^k\}$.

(iii29) $y^2 = t(x^2 - t^k)(x - 1)(x - 2)(x - 3)(x - 4)(x - 5)(x - 6)$.

(iii30) $y^2 = t(x^2 - t)(x^2 + t)(x^2 - t)\{(x - 1)^2 - t^k\}$.

The cases where the stable model is A_1

(viii2) $y^2 = t(x^4 - t)(x^2 + t^{k_1})(x - 1)(x - 2)$, $y^2 = t(x^4 - t)(x^2 + t^{k_1})\{(x - 1)^2 - t^{k_2}\}$.

(viii3) $y^2 = (x^4 - t)(x^2 - t^{k_1})(x - 1)(x - 2)$, $y^2 = (x^4 - t)(x^2 - t^{k_1})\{(x - 1)^2 - t^{k_2}\}$.

(viii4) $y^2 = (x^2 - t^{2k_1})(x^2 - t)(x^2 + t)(x - 1)(x - 2)$, $y^2 = (x^2 - t^{2k_1})(x^2 - t)(x^2 + t)$
$\{(x - 1)^2 - t^{k_2}\}$.

(viii6) $y^2 = (x^2 - t^{k_1})\{(x - 1)^2 - t^{k_2}\}(x - 2)(x - 3)(x - 4)(x - 5)$.

(viii7) $y^2 = t(x^2 - t^{2k_1})(x^2 - t)(x^2 + t)(x - 1)(x - 2)$, $y^2 = t(x^2 - t^{2k_1})(x^2 - t)(x^2 + t)$ $\{(x-1)^2 - t^{k_2}\}$.

(viii8) $y^2 = (x^2 - t)(x^2 + t)\{(x^2 - t)^2 - t^k x^l\}$, $(0 \le l \le 3)$.

(viii9) $y^2 = (x - 1)(x^2 - t)(x - t)\{(x^2 - t)^2 - t^k x^l\}$, $(0 \le l \le 3)$.

(viii10) $y^2 = t(x - 1)(x^2 - t)(x - t)\{(x^2 - t)^2 - t^k x^l\}$, $(0 \le l \le 3)$.

(viii11) $y^2 = t(x^2 - t)(x^2 + t)\{(x^2 - t)^2 - t^k x^l\}$, $(0 \le l \le 3)$.

The cases where the stable model is A_0

(xv3) $y^2 = (x^2 - t^{k_1})\{(x-1)^2 - t^{k_2}\}\{(x-2)^2 - t^{k_3}\}(x - 3)(x - 4)$.

(xv4) $y^2 = t(x^2 - t)\{(x^2 - t)^2 - t^k x^l\}(x - 1)(x - 2)$,
$y^2 = t(x^2 - t)\{(x^2 - t)^2 - t^{k_1} x^l\}\{(x-1)^2 - t^{k_2}\}$.

(xv5) $y^2 = (x^2 - t)\{(x^2 - t)^2 - t^k x^l\}(x - 1)(x - 2)$,
$y^2 = (x^2 - t)\{(x^2 - t)^2 - t^{k_1} x^l\}\{(x-1)^2 - t^{k_2}\}$.

(xv6) $y^2 = tx(x - 1)\{(x^3 - t)^2 - t^k x^l\}$ $(0 \le l \le 5,\ 3k + l \ge 7)$.

(xv8) $y^2 = x(x - 1)\{(x^3 - t)^2 - t^k x^l\}$ $(0 \le l \le 5,\ 3k + l \ge 7)$.

The cases where the stable model is B_{21}

B_{21}: $V_1 = (\mathrm{ii}s_1)$, $V_2 = (\mathrm{ii}s_2)$. $y^2 = g_{s_1}(x, t, k_1) f_{s_2}(x - 1, t, k_2, 0)$.

In the case of B_{21}: $V_1 = (\mathrm{ii}s_1)$, $V_2 = (\mathrm{ii}s_2)$, and the screw number is special (not appearing in the above equation), examples of their equations are as follows (we write (ii2)-(iv2) instead of writing $V_1 = $ (ii2), $V_2 = $ (iv2) for simplicity):

(ii2)-(iv2) $y^2 = t(x^3 - t^2)(x^5 - t^2)$. (ii2)-(iv6) $y^2 = tx(x^2 - 1)(x^5 - t^2)$.

(ii3)-(iv2) $y^2 = t(x^5 - t^2)\{(x-1)^3 - t^2\}$. (ii3)-(iv4) $y^2 = t(x^5 - t^2)\{(x-1)^3 - t\}$.

(ii3)-(iv5) $y^2 = t(x^3 - t^2)(x^5 - t^3)$. (ii3)-(iv6) $y^2 = t(x^5 - t^2)(x - 1)\{(x-1)^2 - t\}$.

(ii3)-(iv8) $y^2 = t(x^5 - t^2)(x - 1)(x - 2)(x - 3)$.

(ii4)-(iv2) $y^2 = t(x^5 - t^4)\{(x-1)^3 - t^2\}$. (ii4)-(iv3) $y^2 = t(x^3 - t)(x^5 - t)$.

(ii4)-(iv4) $y^2 = t(x^5 - t^2)\{(x-1)^3 - t\}$. (ii4)-(iv5) $y^2 = t(x^3 - t)(x^5 - t^3)$.

(ii4)-(iv6) $y^2 = t(x^5 - t^4)(x - 1)\{(x-1)^2 - t\}$. (ii4)-(iv7) $y^2 = tx(x^2 - t)(x^5 - t)$.

(ii4)-(iv8) $y^2 = t(x^5 - t^4)(x - 1)(x - 2)(x - 3)$.

(ii6)-(iv2) $y^2 = tx(x^4 - t^3)\{(x-1)^3 - t^2\}$. (ii6)-(iv4) $y^2 = t(x^4 - t^3)(x^3 - t)(x - 1)$.

(ii6)-(iv5) $y^2 = t(x^4 - t)(x^3 - t^2)(x - 1)$.

(ii6)-(iv6) $y^2 = tx(x^4 - t^3)(x - 1)\{(x-1)^2 - t\}$.

(ii6)-(iv7) $y^2 = tx(x^2 - t)(x^4 - t)(x - 1)$

(ii6)-(iv8) $y^2 = tx(x -^2 - t^3)(x - 1)(x - 2)(x - 3)$.

(ii7)-(iv2) $y^2 = tx(x -^4 - t)\{(x-1)^3 - t^2\}$. (ii7)-(iv4) $y^2 = tx(x -^4 - t)\{(x-1)^3 - t\}$.

(ii7)-(iv5) $y^2 = tx(x^4 - t)(x - 1)\{(x-1)^2 - t\}$.

(ii7)-(iv8) $y^2 = tx(x^4-t)(x-1)(x-2)(x-3)$. (ii9)-(iv2) $y^2 = tx(x^4-t^3)(x^3-t)$.

(ii9)-(iv4) $y^2 = (x^4-t)(x^3-t)(x-1)$.

(ii9)-(iv6) $y^2 = x(x^4-t)(x^2-t)(x-1)$. (ii13)-(iv2) $y^2 = t(x^5-t)\{(x-1)^3-t^2\}$.

(ii13)-(iv4) $y^2 = t(x^5-t)\{(x-1)^3-t\}$.

(ii13)-(iv6) $y^2 = t(x^5-t)(x-1)\{(x-1)^2-t\}$.

(ii13)-(iv8) $y^2 = t(x^5-t)(x-1)(x-2)(x-3)$. (ii15)-(iv2) $y^2 = t(x^5-t^4)(x^3-t)$.

(ii15)-(iv4) $y^2 = (x^5-t)(x^3-t)$. (ii15)-(iv6) $y^2 = t(x^5-t^4)(x^2-t)(x-1)$.

(ii17)-(iv2) $y^2 = t(x^5-t^2)\{(x-1)^3-t\}$. (ii17)-(iv4) $y^2 = t(x^5-t^3)\{(x-1)^3-t\}$.

(ii17)-(iv5) $y^2 = (x^5-t^3)(x^3-t)$. (ii17)-(iv6) $y^2 = t(x^5-t^3)(x-1)\{(x-1)^2-t\}$.

(ii17)-(iv7) $y^2 = (x^5-t^3)(x^2-t)(x-1)$.

(ii17)-(iv8) $y^2 = t(x^5-t^3)(x-1)(x-3)(x-3)$. (ii19)-(iv2) $y^2 = t(x^5-t^2)(x^3-t)$.

(ii20)-(iv2) $y^2 = tx(x^2-t)(x^2+t)\{(x-1)^3-t^2\}$.

(ii20)-(iv4) $y^2 = tx(x^2-t)(x^2+t)\{(x-1)^3-t\}$.

(ii20)-(iv5) $y^2 = (x^3-t^2)(x^2-t)(x^2+t)(x-1)$.

(ii20)-(iv6) $y^2 = tx(x^2-t)(x^2+t)(x-1)\{(x-1)^2-t\}$.

(ii20)-(iv8) $y^2 = tx(x^2-t)(x^2+t)(x-1)(x-2)(x-3)$.

(ii21)-(iv2) $y^2 = tx(x^2-t)(x^2+t)(x^3-t)$. (ii24)-(iv2) $y^2 = tx(x^2-t)(x^2+t)(x^3-t)$.

(ii24)-(iv4) $y^2 = t(x^3-t)(x-1)(x-2)(x-3)(x-4)(x-5)$.

(ii24)-(iv6) $y^2 = tx(x^2-t)(x-1)(x-2)(x-3)(x-4)(x-5)$.

The cases where the stable model is B_{20}

B_{20}: $V_1 = (\mathrm{ii}s_1)$, $V_2 = (\mathrm{xi}s_2)$ $y^2 = g_{s_1}(x,t,k_1)h_{s_2}(x-1,t,k_2,l)$. In the case of special values of the screw number at e_1, examples of their equations are as follows:

(ii6)-(xi2) $y^2 = tx(x^4-t^3)\{(x-1)^2-t^l\}(x-2)$.

(ii13)-(xi2) $y^2 = t(x^5-t)\{(x-1)^2-t^l\}(x-2)$.

(ii17)-(xi2) $y^2 = t(x^5-t^3)\{(x-1)^2-t^l\}(x-2)$.

(ii20)-(xi2) $y^2 = tx(x^2-t)(x^2+t)\{(x-1)^2-t^l\}(x-2)$.

The cases where the stable model is B_{11}

B_{11}: $V_1 = (\mathrm{vii}s_1)$, $V_2 = (\mathrm{iv}s_2)$. $y^2 = f_{s_2}(x-1,t,k,0)\sigma_{s_1}(x,t,k_1,l)$.

The cases where the stable model is B_{10}

B_{10}: $V_1 = (\mathrm{vii}s_1)$, $V_2 = (\mathrm{xi}s_2)$. $y^2 = \sigma_{s_1}(x-1,t,k_1,l_1)h_{s_2}(x,t,k_2,l_2)$.

The cases where the stable model is B_{01}

B_{01}: $V_1 = (\mathrm{xvi}s_1)$, $V_2 = (\mathrm{iv}s_2)$. $y^2 = \tau_{s_1}f_{s_2}(x-1,t,k_3,0)$.

The cases where the stable model is B_{00}

B_{00}: $V_1 = (\text{xvi}s_1)$, $V_2 = (\text{xi}s_2)$. $\quad y^2 = \tau_{s_1} h_{s_2}(x-1, t, k_3, 0)$.

The cases where the stable model is C_{111}

C_{111}: Id, $V_1 = (\text{iv}s_1)$, $V_2 = (\text{iv}s_2)$, $V_3 = (\text{va1})$.
$y^2 = f_{s_1}(x, t, k_1+1, 0) f_{s_2}(x-1, t, k_2+1, 0)(x-2)(x-3)$.

C_{111}: Id, $V_1 = (\text{iv}s_1)$, $V_2 = (\text{iv}s_2)$, $V_3 = (\text{va2})$.
$y^2 = (x^2 - t^3) f_{s_1}(x, t, k_1+1, 1) f_{s_2}(x-1, t, k_2, 0)$ $(0 \leq k_2)$.

C_{111}: Id, $V_1 = (\text{iv}s_1)$, $V_2 = (\text{iv}s_2)$, $V_3 = (\text{va3})$.
$y^2 = (x^2 - t) f_{s_1}(x, t, k_1, 1) f_{s_2}(x-1, t, k_2, 0)$.

C_{111}: Id, $V_1 = (\text{iv}s_1)$, $V_2 = (\text{iv}s_2)$, $V_3 = (\text{va6})$.
$y^2 = (x^2 - t^2) f_{s_1}(x, t, k_1+1, 0) f_{s_2}(x-1, t, k_2, 0)$.

C_{111}: II(1,1), $V_1 = V_3 = (\text{iv}s_1)$, $V_2 = (\text{vb3})$.
$y^2 = tx(x-1)\theta'_{s_1}(x, t, k)$

C_{111}: II(1,1) $V_1 = V_3 = (\text{iv}s_1)$, $V_3 = (\text{vb4})$.
$y^2 = x(x-1)\theta_{s_1}(x, t, k)$.

C_{111}: II(1,1) $V_1 = V_3 = (\text{iv}s_1)$, $V_3 = (\text{vb5})$.
$y^2 = (x^2 - 2t - t^2 x)\theta'_{s_1}(x, t, k)$.

C_{111}: II(1,1) $V_1 = V_3 = (\text{iv}s_1)$, $V_3 = (\text{vb6})$.
$y^2 = t(x^2 - 2t - t^2 x)\theta'_{s_1}(x, t, k)$.

The cases where the stable model is C_{110}

C_{110}: Id, $V_1 = (\text{iv}s_1), V_3 = (\text{iv}s_2)$, $V_3 = (\text{xiia1})$.
$y^2 = f_{s_1}(x, t, k_1+1, 0) f_{s_2}(x-2, t, k_2+1, 0)\{(x-1)^2 - t^{k_3}\}$.

C_{110}: Id, $V_1 = (\text{iv}s_1), V_3 = (\text{iv}s_2)$, $V_3 = (\text{xiia2})$.
$y^2 = t f_{s_1}(x, t, k_1, 1) f_{s_2}(x-2, t, k_2, 1)\{(x-1)^2 - t^{k_3}\}$.

C_{110}: II(1,1) $V_1 = V_2 = (\text{iv}s_1)$, $V_3 = (\text{xiib1})$.
$y^2 = t(x^2 - t^{k_1+1})\theta'_{s_1}$.

C_{110}: II(1,1) $V_1 = V_2 = (\text{iv}s_1)$, $V_3 = (\text{xiib2})$.
$y^2 = (x^2 - t^{k_1+1})\theta'_{s_1}$.

The cases where the stable model is C_{101}

C_{101}: Id, $V_1 = (\text{iv}s_1)$, $V_2 = (\text{xi}s_2)$, $V_3 = (\text{va1})$.
$y^2 = f_{s_1}(x, t, k_1+1, 0) h_{s_2}(x-1, t, k_2, l+1)(x-2)(x-3)$.

C_{101}: Id, $V_1 = (\text{iv}s_1)$, $V_2 = (\text{xi}s_2)$, $V_3 = (\text{va2})$.
$y^2 = (x^2 - t^3) h_{s_2}(x, t, k_2, l - 1/2) f_{s_1}(x-1, t, k_1, 0)$.

C_{101}: Id, $V_1 = (\text{iv}s_1)$, $V_2 = (\text{xi}s_2)$, $V_3 = (\text{va3})$.
$y^2 = (x^2 - t) h_{s_2}(x, t, k_2, l+1) f_{s_1}(x-1, t, k_1, 0)$.

C_{101}: Id, $V_1 = (\mathrm{iv}s_1)$, $V_2 = (\mathrm{xi}s_2)$, $V_3 = (\mathrm{va6})$.
$y^2 = (x^2 - t^2)h_{s_2}(x, t, k_2, l+1)f_{s_1}(x-1, t, k_1, 0)$.

The cases where the stable model is C_{100}

C_{100}: Id $V_1 = (\mathrm{iv}s_1)$, $V_2 = (\mathrm{xi}s_2)$, $V_3 = (\mathrm{xiia1})$.
$y^2 = f_{s_1}(x-1, t, k_1+1, 0)h_{s_2}(x, t, k_2, l)\{(x-2)^2 - t^{k_3}\}$.
C_{100}: Id $V_1 = (\mathrm{iv}s_1)$, $V_2 = (\mathrm{xi}s_2)$, $V_3 = (\mathrm{xiia2})$.
$y^2 = f_{s_1}(x-1, t, k_1, 1)h_{s_2}(x, t, k_2, l-1/2)\{(x-2)^2 - t^{k_3}\}$.

The cases where the stable model is C_{001}

C_{001}: Id, $V_1 = (\mathrm{xi}s_1)$, $V_2 = (\mathrm{xi}s_2)$, $V_3 = (\mathrm{va1})$.
$y^2 = h_{s_1}(x-1, t, k_1, l)h_{s_2}(x, t, k_2, l)(x-2)(x-3)$.
C_{001}: Id, $V_1 = (\mathrm{xi}s_1)$, $V_2 = (\mathrm{xi}s_2)$, $V_3 = (\mathrm{va2})$.
$y^2 = t(x^2 - t)h_{s_1}(x, t, k_1, l)h_{s_2}(x-1, t, k_2, l-1/2)$.
C_{001}: Id, $V_1 = (\mathrm{xi}s_1)$, $V_2 = (\mathrm{xi}s_2)$, $V_3 = (\mathrm{va3})$.
$y^2 = (x^2 - t)h_{s_1}(x, t, k_1, l-1/2)h_{s_2}(x-1, t, k_2, l)$.
C_{001}: Id, $V_1 = (\mathrm{xi}s_1)$, $V_2 = (\mathrm{xi}s_2)$, $V_3 = (\mathrm{va5})$.
$y^2 = (x^2 - t^2)h_{s_1}(x, t, k_1, l)h_{s_2}(x-1, t, k_2, l)$.
C_{001}: II(1,1), $V_1 = V_2 = (\mathrm{xi1})$, $V_3 = (\mathrm{vb3})$. $y^2 = tx(x-1)\widetilde{F}_{1,2}^{2l-1,k+4l-2}$.
C_{001}: II(1,1), $V_1 = V_2 = (\mathrm{xi2})$, $V_3 = (\mathrm{vb3})$ $y^2 = tx(x-1)\widetilde{F}_{1,2}^{2l-2,k+4l-4}$.
C_{001}: II(1,1), $V_1 = V_2 = (\mathrm{xi1})$, $V_3 = (\mathrm{vb4})$. $y^2 = tx(x-1)\widetilde{F}_{1,2}^{2l-1,k+4l-2}$.
C_{001}: II(1,1), $V_1 = V_2 = (\mathrm{xi2})$, $V_3 = (\mathrm{vb4})$. $y^2 = tx(x-1)\widetilde{F}_{1,2}^{2l-2,k+4l-4}$.
C_{001}: II(1,1), $V_1 = V_2 = (\mathrm{xi1})$, $V_3 = (\mathrm{vb5})$. $y^2 = \{(x^2 - 2t) - t^2x\}\widetilde{F}_{1,2}^{2l,k+4l}$.
C_{001}: II(1,1), $V_1 = V_2 = (\mathrm{xi2})$, $V_3 = (\mathrm{vb5})$. $y^2 = \{(x^2 - 2t) - t^2x\}\widetilde{F}_{1,2}^{2l-1,k+4l-2}$.
C_{001}: II(1,1), $V_1 = V_2 = (\mathrm{xi1})$, $V_3 = (\mathrm{vb6})$. $y^2 = t\{(x^2 - 2t) - t^2x\}\widetilde{F}_{1,2}^{2l,k+4l}$.
C_{001}: II(1,1), $V_1 = V_2 = (\mathrm{xi2})$, $V_3 = (\mathrm{vb6})$. $y^2 = t\{(x^2 - 2t) - t^2x\}\widetilde{F}_{1,2}^{2l-1,k+4l-2}$.

The cases where the stable model is C_{000}

C_{000}: Id, $V_1 = (\mathrm{xi}s_1)$, $V_2 = (\mathrm{xi}s_2)$, $V_3 = (\mathrm{xiia1})$.
$y^2 = h_{s_1}(x, t, k_1, l)h_{s_2}(x-1, t, k_2, l)\{(x-3)^2 - t^k\}$.
C_{000}: Id, $V_1 = (\mathrm{xi}s_1)$, $V_2 = (\mathrm{xi}s_2)$, $V_3 = (\mathrm{xiia2})$.
$y^2 = th_{s_1}(x, t, k_1, l-1/2)h_{s_2}(x-1, t, k_2, l-1/2)\{(x-3)^2 - t^k\}$.
C_{000}: II(1,1), $V_1 = V_2 = (\mathrm{xi1})$, $V_3 = (\mathrm{xiib1})$. $y^2 = t(x^2 - t^{k_1+2})\widetilde{F}_{12}^{2l,k_2+4l}$.
C_{000}: II(1,1), $V_1 = V_2 = (\mathrm{xi2})$, $V_3 = (\mathrm{xiib1})$. $y^2 = t(x^2 - t^{k_1+2})\widetilde{F}_{12}^{2l+1,k_2+4l}$.
C_{000}: II(1,1), $V_1 = V_2 = (\mathrm{xi1})$, $V_3 = (\mathrm{xiib2})$. $y^2 = (x^2 - t^{k_1+2})\widetilde{F}_{12}^{2l,k_2+4l}$.
C_{000}: II(1,1), $V_1 = V_2 = (\mathrm{xi2})$, $V_3 = (\mathrm{xiib2})$. $y^2 = (x^2 - t^{k_1+2})\widetilde{F}_{12}^{2l+1,k_2+4l}$.

The cases where the stable model is E_{11}

E_{11}: Id, $V_1 = (\text{va}s_1)$, $V_2 = (\text{va}s_2)$. $\quad y^2 = \omega_{s_1}(x,t,k_1)\omega_{s_2}(x-1,t,k_2)$.

E_{11}: II(0,1), $V_1 = (\text{vb}s_1)$, $V_2 = (\text{vb}s_2)$. $\quad y^2 = t\Gamma_{s_1}(x,t,k_1)\Gamma_{s_2}(x-1,t,k_2)$.

We have to give more examples when the screw number at e_1 is special.

E_{11}: II(0,1), $V_1 = (\text{vb}1)$,$V_2 = (\text{vb}4)$. $\quad y^2 = x(x^4-t)(x^3-t)$.

E_{11}: II(0,1), $V_1 = V_2 = (\text{vb}2)$. $\quad y^2 = x(x^3-t^2)(x^3-t)(x-1)$.

E_{11}: II(0,1), $V_1 = (\text{vb}2)$,$V_2 = (\text{vb}4)$. $\quad y^2 = (x^4-t^3)(x^3-t)(x-1)$.

E_{11}: II(0,1), $V_1 = (\text{vb}2)$, $V_2 = (\text{vb}6)$. $\quad y^2 = (x^4-t^2)(x^3-t)(x-1)$.

E_{11}: II(1,2), $V_1 = V_2 = (\text{va}s)$. $\quad y^2 = t\omega'_s$.

E_{11}: II(1,3), $V_1 = V_2 = (\text{va}s)$. $\quad y^2 = \omega'_s$.

The cases where the stable models is E_{10}

E_{10}: Id, $V_1 = (\text{va}s_1)$, $V_2 = (\text{xiia}s_2)$. $\quad y^2 = \omega_{s_1}(x,t,k_1)\rho_{s_2}(x-2,t,k_2,l)$.

E_{10}: II(0,1), $V_1 = (\text{vb}s_1)$, $V_2 = (\text{xiib}s_2)$. $\quad y^2 = t\Gamma_{s_1}(x,t,k_1)\rho'_{s_2}(x-1,t,k_2,l)$.

We have to give more examples when the screw number at e_1 is special.

$V_1 = (\text{vb}2)$, $V_2 = (\text{xiib}2)$ $\quad y^2 = x(x^3-t^2)(x^2-t)\{(x-1)^2-t^{k-1}\}$.

$V_1 = (\text{vb}4)$, $V_2 = (\text{xiib}2)$ $\quad y^2 = x(x^3-t^2)(x^2-t)\{(x-1)^2-t^{k-1}\}$.

The cases where the stable model is E_{00}

E_{00}: Id, $V_1 = (\text{xiia}s_1)$, $V_2 = (\text{xiia}s_2)$. $\quad y^2 = \rho_{s_1}(x,t,k_1,l_1)\rho_{s_2}(x-1,t,k_2,l_2)$.

E_{00}: II(0,1), $V_1 = (\text{xiib}s_1)$, $V_2 = (\text{xiib}s_2)$. $\quad y^2 = \rho'_{s_1}(x,t,k_1,l_1)\rho'_{s_2}(x-1,t,k_2,l_2)$.

E_{00}: II(1,2), $V_1 = V_2 = (\text{xiia}1)$.

$y^2 = t\{(x^2-t)^2 - t^{k_1}x^{l_1}\}\{(x^2-2t)^2 - t^{k_2}x^{l_2}\}$, $(2k_i + l_i - 4 \geq 1,\ i = 1,2)$.

E_{00}: II(1,2), $V_1 = V_2 = (\text{xiia}2)$.

$y^2 = t\{(x^2-t)^2 - t^{k_1}x^{l_1}\}\{(x^2-2t)^2 - t^{k_2}x^{l_2}\}$, $(2k_1 + l_1 - 4 = 2l-1,\ 2k_2 + l_2 - 4 = k + 2l)$.

E_{00}: II(1,3), $V_1 = V_2 = (\text{xiia}1)$.

$y^2 = \{(x^2-t)^2 - t^{k_1}x^{l_1}\}\{(x^2-2t)^2 - t^{k_2}x^{l_2}\}$, $(2k_1 + l_1 - 4 = 2l,\ 2k_2 + l_2 - 4 = k + 2l)$.

E_{00}: II(1,3), $V_1 = V_2 = (\text{xiia}2)$.

$y^2 = \{(x^2-t)^2 - t^{k_1}x^{l_1}\}\{(x^2-2t)^2 - t^{k_2}x^{l_2}\}$, $(2k_1 + l_1 - 4 = 2l-1,\ 2k_2 + l_2 - 4 = 2l + k)$.

The cases where the stable model is F_{11}

F_{11}: Id, $V_1 = (\text{iv}s_1)$, $V_2 = (\text{iv}s_2)$.

$y^2 = (x-\alpha t^l)(x-2)f_{s_2}(x,t,k_1+1,l-1)f_{s_1}(x-1,t,k_2+1,0)$.

F_{11}: II(0,1), $V_1 = (\text{iv}s_1)$, $V_2 = (\text{iv}s_2)$.

$y^2 = t(x-\alpha t^l)(x-2)f_{s_2}(x,t,k_1+1,l-1)f_{s_1}(x-1,t,k_2+1,0)$.

F_{11}: II(2,1), $V_1 = V_2 = (\text{iv}s)$. $\quad y^2 = \eta_s$.

F_{11}: II(2,2), $V_1 = V_2 = (\text{iv}s)$. $\quad y^2 = t\eta_s$.

The cases where the stable model is F_{10}

F_{10}: Id, $V_1 = (\text{iv}s_1)$, $V_2 = (\text{xi}s_2)$.
$y^2 = (x - t^{l_1}) f_{s_1}(x, t, k_1, 0) h_{s_2}(x - 1, t, k_2 + 1, l + l_1/2)(x - 2)$.
F_{10}: II(0,1), $V_1 = (\text{iv}s_1)$, $V_2 = (\text{xi}s_2)$.
$y^2 = (x - t^{l_1}) f_{s_1}(x, t, k_1, l_1) h_{s_2}(x - 1, t, k_2, l - 1/2)(x - 2)$.

The cases where the stable model is F_{00}

F_{00}: Id, $V_1 = (\text{xi}s_1)$, $V_2 = (\text{xi}s_2)$.
$y^2 = (x - t^{l_1}) f_{s_1}(x, t, k_1, l) h_{s_2}(x - 1, t, k_2, l)(x - 2)$.
F_{00}: II(0,1), $V_1 = (\text{xi}s_1)$, $V_2 = (\text{xi}s_2)$.
$y^2 = (x - t^{l}) h_{s_1}(x, t, k_1, l_1 + l/2) h_{s_2}(x - 1, t, k_2, l - 1/2)(x - 2)$.
F_{00}: II(2,1), $V_1 = V_2 = (\text{xi}1)$. $y^2 = \widetilde{F}_{12}^{2l_1+l+1,k+4(l_1+l)+2}(x^2 - t^{k_2}x^{l_2})$, $(2_{k_2} + l_2 - 4 = l + 1)$.
F_{00}: II(2,1), $V_1 = V_2 = (\text{xi}2)$. $y^2 = \widetilde{F}_{12}^{2l_1+l+1,k+4(l_1+l)}(x^2 - t^{k_2}x^{l_2})$, $(2_{k_2} + l_2 - 4 = l + 1)$.
F_{00}: II(2,2), $V_1 = V_2 = (\text{xi}1)$. $y^2 = t\widetilde{F}_{12}^{2l_1+l+1,k+4(l_1+l)+2}(x^2 - t^{k_2}x^{l_2})$, $(2_{k_2} + l_2 - 4 = l + 1)$.
F_{00}: II(2,2), $V_1 = V_2 = (\text{xi}2)$. $y^2 = t\widetilde{F}_{12}^{2l_1+l+1,k+4(l_1+l)}(x^2 - t^{k_2}x^{l_2})$, $(2_{k_2} + l_2 - 4 = l + 1)$.

The cases where the stable model is G_{11}

G_{11}: Id, $V_1 = (\text{va}s_1)$, $V_3 = (\text{iv}s_2)$. $y^2 = f_{s_2}(x, t, k_1 + 1, 0)(x - 1)\omega_{s_1}(x - 2, t, k_2 + 1)$.
G_{11}: II(0,1), $V_1 = (\text{vb}s_1)$, $V_3 = (\text{iv}s_2)$. $y^2 = t\Gamma_{s_1}(x, t, k_1)(x - 1) f_{s_2}(x, t, k_2, 1)$.

The cases where the stable model is G_{10}

G_{10}: Id, $V_1 = (\text{va}s_1)$, $V_3 = (\text{xi}s_2)$. $y^2 = \omega_{s_1}(x, t, k_1 + 1)(x - 1) h_{s_2}(x - 2, t, k_2, l)$.
G_{10}: II(0,1), $V_1 = (\text{vb}s_1)$, $V_3 = (\text{xi}s_2)$. $y^2 = \Gamma_{s_1}(x, t, k_1 + 1)(x - 1) h_{s_2}(x - 2, t, k_2, l - 1/2)$.

The cases of the stable models are G_{00}

G_{00}: Id, $V_1 = (\text{xiia}s_1)$, $V_3 = (\text{xi}s_2)$. $y^2 = \rho_{s_1}(x, t, k_1, l + 1)(x - 1) h_{s_2}(x - 2, t, k_2, l)$.
G_{00}: II(0,1), $V_1 = (\text{xiia}s_1)$, $V_3 = (\text{xi}s_2)$. $y^2 = t\rho'_{s_1}(x, t, k_1, l)(x - 1) h_{s_2}(x - 2, t, k_2, l - 1/2)$.

The cases of the stable models are G_{01}

G_{00}: Id, $V_1 = (\text{xiia}s_1)$, $V_3 = (\text{iv}s_2)$. $y^2 = \rho_{s_1}(x, t, k_1, l + 1)(x - 1) f_{s_2}(x - 2, t, k_2, l)$.
G_{00}: II(0,1), $V_1 = (\text{xiia}s_1)$, $V_3 = (\text{iv}s_2)$. $y^2 = t\rho'_{s_1}(x, t, k_1, l)(x - 1) f_{s_2}(x - 2, t, k_2, l - 1/2)$.

The cases where the stable model is J_1

J_1: Id, $V_1 = (\text{va}s)$. $y^2 = \omega_s(x, t, k_1 + 1)\{(x - 1)^2 - t^{k_2}\}\{(x - 2)^2 - t^{k_3}\}$.
J_1: II(0,1), $V_1 = (\text{va}s)$. $y^2 = \omega_s(x, t, k_1 + 1)\{(x^2 - t)^2 - t^{k_2}x^l\}$, $(2k_2 + l - 4 \geq 1)$.
J_1: II(1,4), $V_1 = (\text{vb}s)$. $y^2 = t\Gamma_s(x - 1, t, k_1)\{(x^2 - t)^2 - t^{k_2}x^l\}$, $(2k_2 + l - 4 \geq 1)$.
J_1: II(1,6), $V_1 = (\text{vb}s)$. $y^2 = t\Gamma_s(x, t, k_1)\{(x - 1)^2 - t^{k_2}\}\{(x - 2)^2 - t^{k_3}\}$.

The cases where the stable models is J_0

J_0: Id, $V_1 = (\text{xiia}s)$. $\quad y^2 = \rho_s(x,t,k_1+1)\{(x-1)^2 - t^{k_2}\}\{(x-2)^2 - t^{k_3}\}$.
J_0: II(0,1), $V_1 = (\text{xiia}s)$. $\quad y^2 = \rho_s(x,t,k_1+1)\{(x^2-t)^2 - t^{k_2}x^l\}$ $(2k_2+l-4 \geq 1)$.
J_0: II(1,4), $V_1 = (\text{xiib}s)$. $\quad y^2 = t\rho'_s(x-1,t,k_1)\{(x^2-t)^2 - t^{k_2}x^l\}$ $(2k_2+l-4 \geq 1)$.
J_0: II(1,6), $V_1 = (\text{xiib}s)$. $\quad y^2 = t\rho'_s(x,t,k_1)\{(x-1)^2 - t^{k_2}\}\{(x-2)^2 - t^{k_3}\}$.

The cases where the stable model is K_1

K_1: Id, $V_4 = (\text{iv}s)$, $\quad y^2 = (x-t)f_s(x,t,k_1+1,1)\{(x-1)^2 - t^{k_2}\}\{(x-2)^2 - t^{k_3}\}$.
K_1: II(0,1), $V_4 = (\text{iv}s)$.
$y^2 = \{(x^2-t)^2 - t^{k_1}x^{l_1}\}(x-1)f_s(x-2,t,k+1,0)$, $(2k_1+l_1-4 \geq 1)$.
K_1: II(1,4), $V_4 = (\text{iv}s)$.
$y^2 = t\{(x^2-t)^2 - t^{k_1}x^{l_1}\}\{(x-1) - t^{2l_2-2}\}f_s(x-1,t,k_2+l_2-1,1)$, $(2k_1+l_1-4 \geq 1)$.
K_1: II(1,6), $V_4 = (\text{iv}s)$. $\quad y^2 = t(x^2-t^{k_1})\{(x-1)^2-t^{k_2}\}\{(x-2)-t^{2l}\}f_s(x-3,t,k_3+l,1)$.

The cases where the stable model is K_0

K_0: Id, $V_4 = (\text{xi}s)$. $\quad y^2 = (x-t)h_s(x,t,k_1,l)\{(x-1)^2 - t^{k_2}\}\{(x-2)^2 - t^{k_3}\}$.
K_0: II(0,1), $V_4 = (\text{xi}s)$.
$y^2 = \{(x^2-t)^2 - t^{k_1}x^{l_1}\}(x-1)f_s(x-2,t,k,l)$ $(2k_1+l_1-4 \geq 1)$
K_0: II(1,4), $V_4 = (\text{iv}s)$.
$y^2 = t\{(x^2-t)^2 - t^{k_1}x^{l_1}\}\{(x-1) - t^{2l_2-2}\}h_s(x-1,t,k_2,l_2-1/2)$ $(2k_1+l_1-4 \geq 1)$.
K_0: II(1,6), $V_4 = (\text{iv}s)$.
$y^2 = t(x^2 - t^{k_1})\{(x-1)^2 - t^{k_2}\}\{(x-2) - t^{2l}\}h_s(x-3,t,k_3,l-1/2)$.

The cases where the stable model is L

L: II(0,1), $V_1 = V_2 = (\text{xb})$. $\quad y^2 = (x^2 - t^{k_1+1})\{(x^2-t)^2 - t^{k_2}x^l\}\{(x-1)^2 - t^{k_3})\}$.
L: II(0,2), $V_1 = V_2 = (\text{xc})$. $\quad y^2 = \{(x^2-t)^2 - t^{k_1}x^{l_1}\}\{(x^2-t)^2 - t^{k_2}x^{l_2}\}$.
L: II(1,2). $\quad y^2 = t\{(x^2-t)^2 - t^{k_1}x^{l_1}\}\{x^2 - t^{l_2+1}\}\{(x-1)^2 - t^{l_3-1}\}$.
L: II(1,5). $\quad y^2 = t\{(x^2-t)^2 - t^{k_1}x^{l_1}\}\{(x^2-t)^2 - 2t^{k_2}x^{l_2}\}$.
L: II(1,8). $\quad y^2 = t(x^2 - t^{k_1})\{(x-1)^2 - t^{k_2}\}\{(x-2)^2 - t^{k_3}\}\{(x-3)^2 - t^{k_4}\}$.
L: III(0,1), $V_1 = (\text{vd}s)$ $\quad y^2 = \{(x^3-t)^2 - t^{k_1}x^{l_1}\}\{(x-1)^2 - t^{k_2-1}\}$ $(3k_1+2l_1-6 \geq 0)$.
$y^2 = \{(x^3-t)^2 - t^{k_1}x^{l_1}\}\{x^2 - t^{k_2+1}\}$ $(3k_1+2l_1-6 \geq 0)$.
L: IV(0,1), $V_1 = (\text{ve})$. $\quad y^2 = \{(x^4-t)^2 - t^{k_1}x^{l_1}\}$ $(3k_1+3l_1-8 \geq 0)$.
L: VI(1,1), $V_1 = (\text{vd}s)$. $\quad y^2 = t\{(x^3-t)^2 - t^{k_1}x^{l_1}\}\{(x-1)^2 - t^{k_2-1}\}$ $(3k_1+2l_1-6 \geq 0)$.

The cases where the stable model is N

N: II(0,1).
$y^2 = \{(x^2-t)^2 - t^{k_1}x^{l_1}\}\{(x-1)^2 - t^{k_2+2(l_2-1)}\}\{(x-1-t^{l_2-1})^2 - t^{k_3+2(l_2-1)}\}$.

N: II(0,2). $\quad y^2 = \{(x^2 - t^{2l-1})^2 - t^{k_1+2(l-1)}x^{l_1}\}\{(x^2 - 2x + 1 - t)^2 - t^{k_2}x^{l_2}\}.$

N: II(2,3). $\quad y^2 = t\{(x^2 - t)^2 - t^{k_1}x^{l_1}\}[\{(x-1)^2 - t\}^2 - t^{k_2}x^{l_2}].$

N: II(2,4).

$y^2 = t\{(x - t^{k_1})^2 - t^{2(k_1+1)+k_2}\}\{x^2 - t^{2(k_1+1)+k_3}\}\{(x-1)^2 - t^{k_4}\}\{(x-2)^2 - t^{k_5}\}.$

N: II(2,8).

$y^2 = \{((x^2 - t) - t^{k_1}x^{l_1})^2 - t^{k_2}x^{l_2}\}\{(x^2 - t)^2 - t^{k_3}x^{l_3}\},$

$(2k_1 + l_1 - 2 = l, 2k_2 + l_2 - 4 = K_1 + 2l, 2k_3 + l_3 - 4 = K_2 + 2l,).$

N: IV(2,2).

$y^2 = (x^2 - t)^4 - 2(x^2 - t)^2 x^{p_1} t^{p_2} + x^{q_1} t^{q_2} + (x^2 - t)^L x^{m_1} t^{m_2}$. ($L \leq 3,\ 2p_1 + 4p_2 - 8 = q_1 + 2q_2 - 8 = 4l + 1,\ m_1 + 2m_2 - 2 = k_1 + 4l - l_1,\ 2k_1 + l_1 - 4$ is positive even integer.)

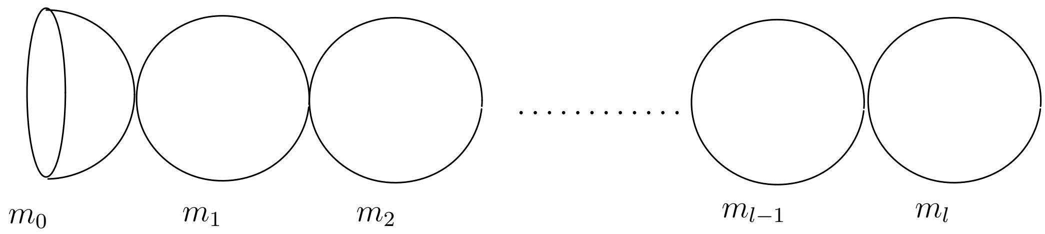


Figure A

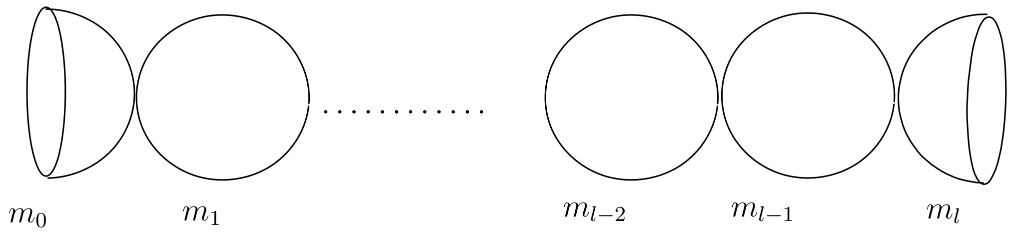


Figure B

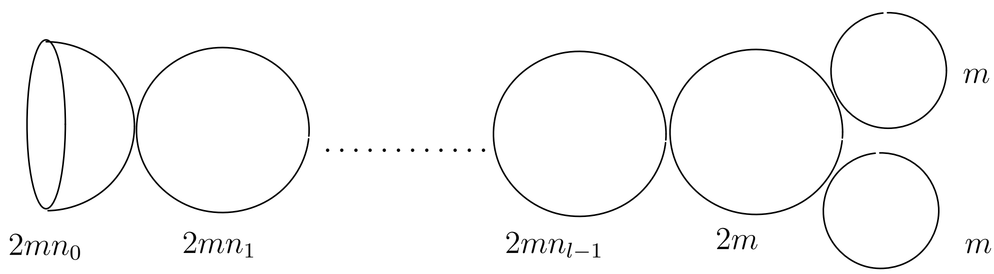


Figure C

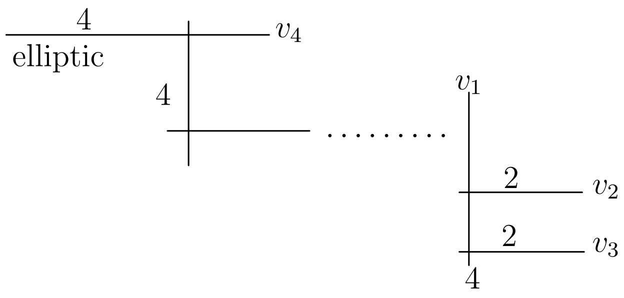


Figure 1

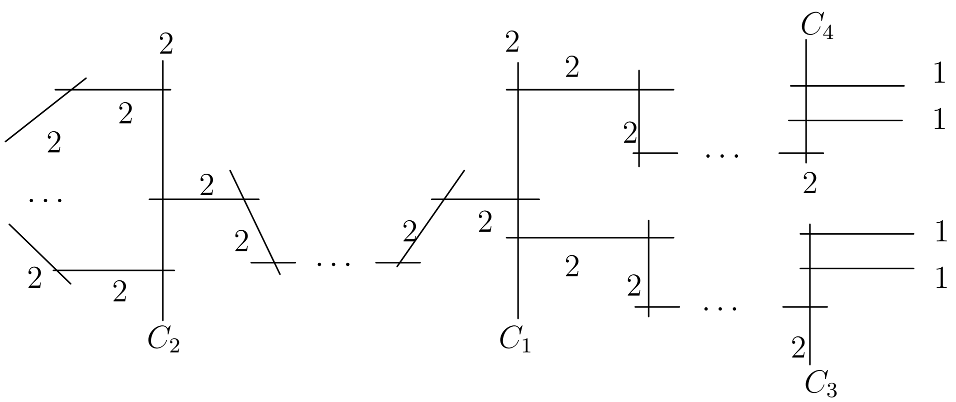


Figure 2

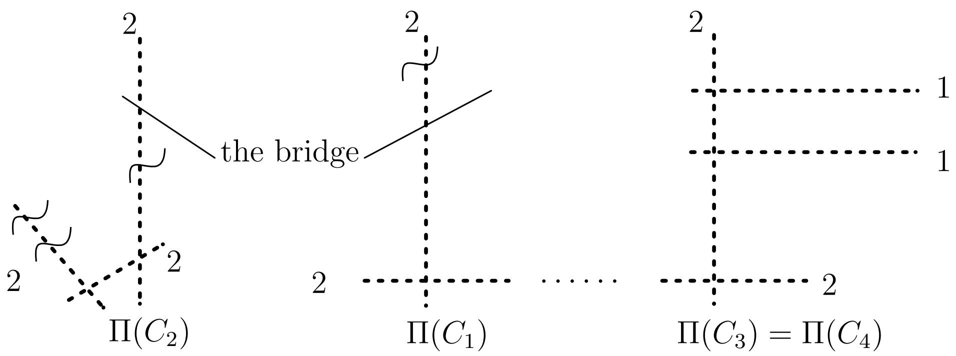


Figure 3

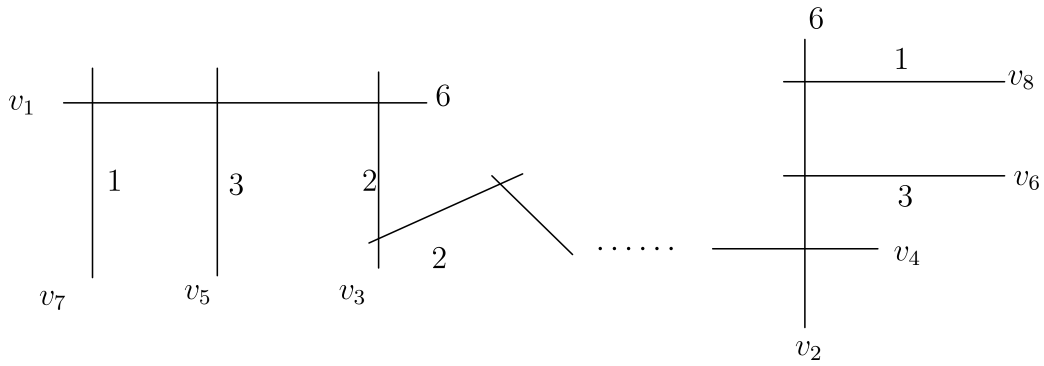


Figure 4

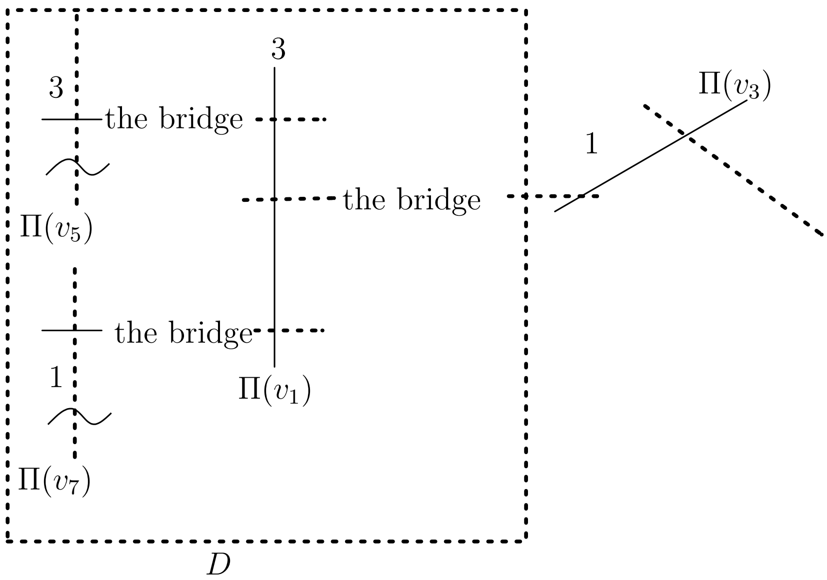
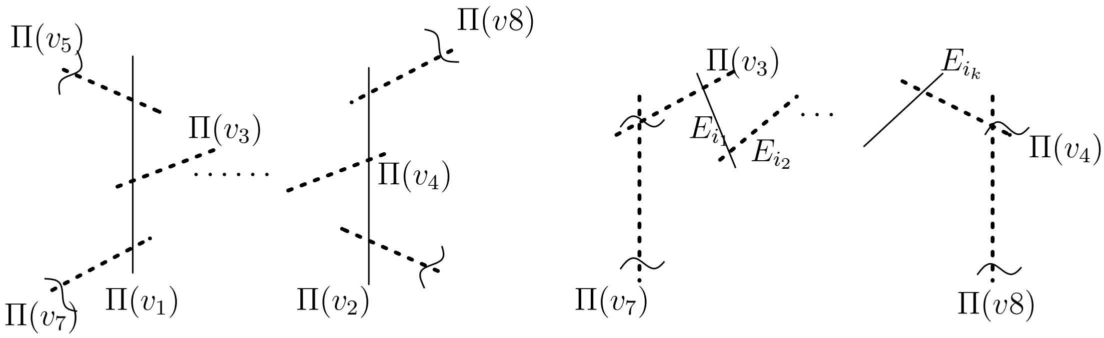


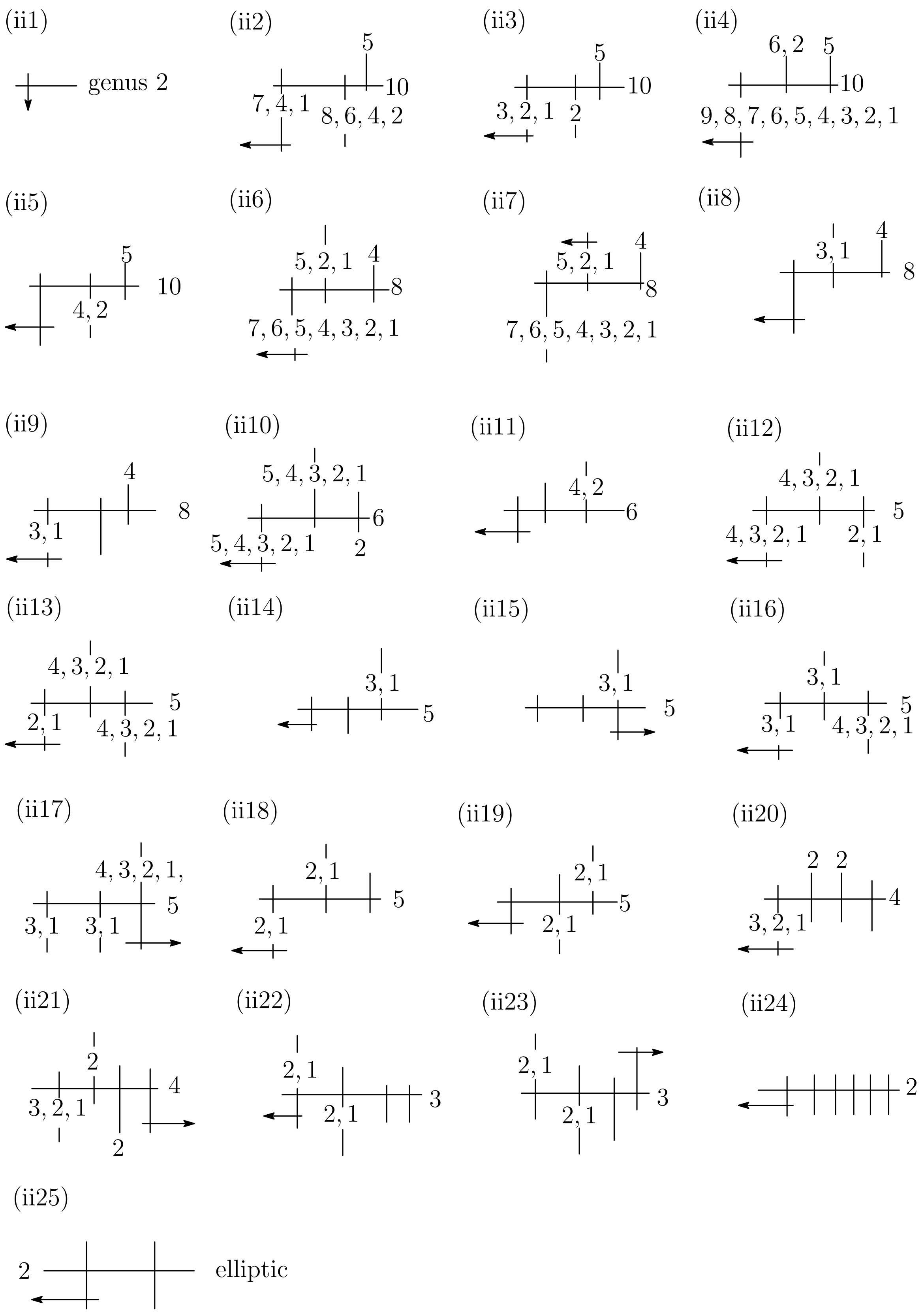
Figure 5

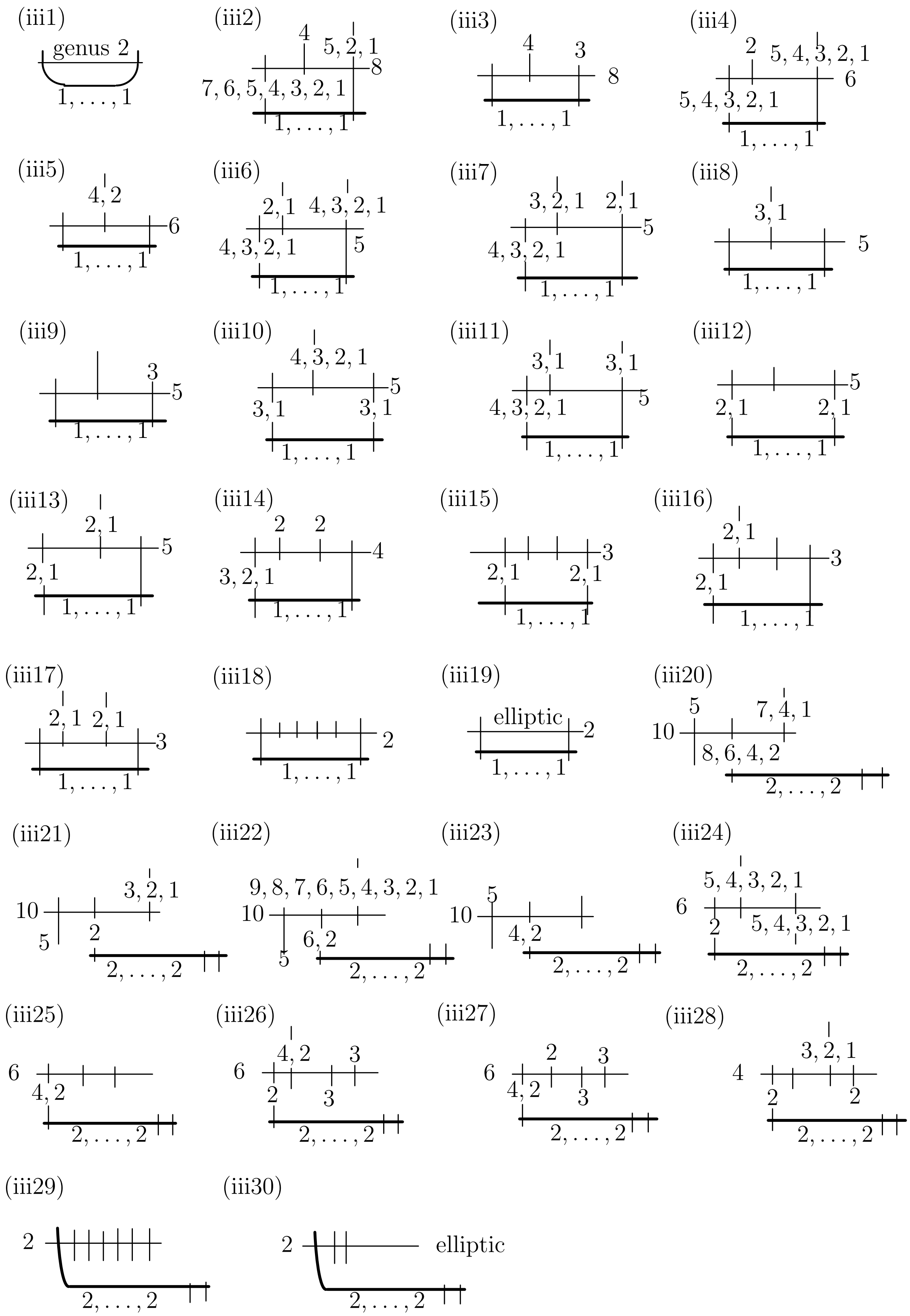


$\Pi(v_5)$
$\Pi(v_3)$
$\Pi(v_7)$
$\Pi(v_1)$
$\Pi(v8)$
$\Pi(v_4)$
$\Pi(v_2)$
$\Pi(v_3)$
E_{i_1}
E_{i_2}
E_{i_k}
$\Pi(v_4)$
$\Pi(v_7)$
$\Pi(v8)$

Figure 6

Table 1 Marked generalized quotient

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8
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(iii4)
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1, . . . , 1
(iii6)
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(iii7)
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1, . . . , 1
(iii8)
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(iii17)
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(iii30)
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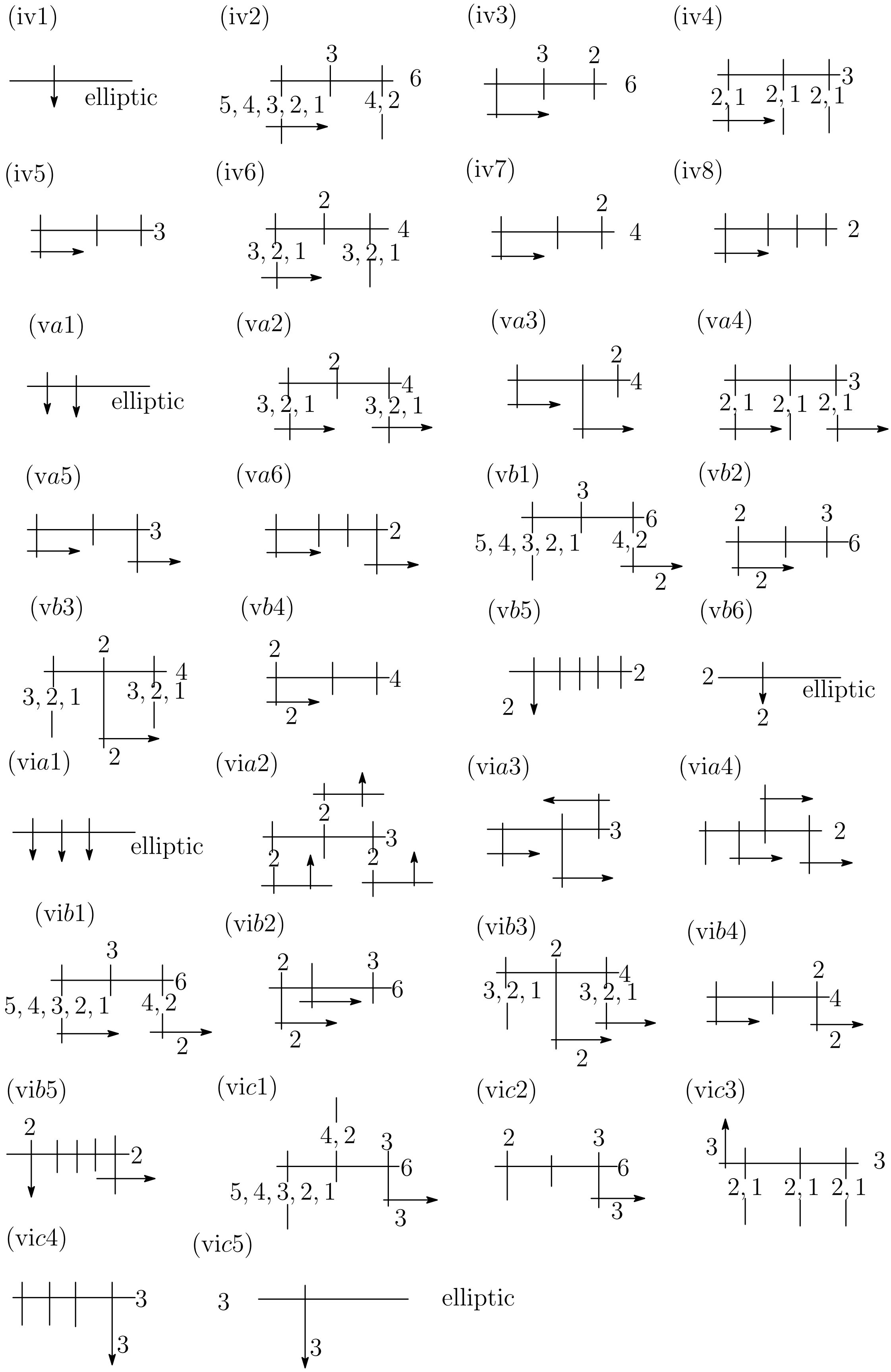
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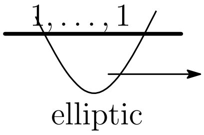
(vic5)
3
elliptic
3

(vii1)



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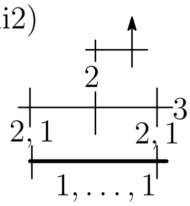

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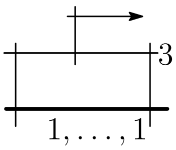

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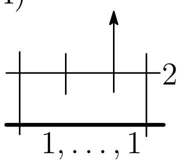

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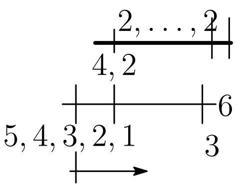

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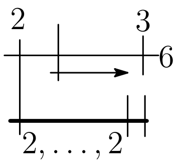

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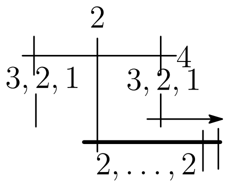

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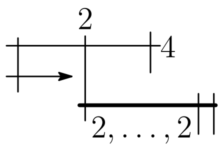

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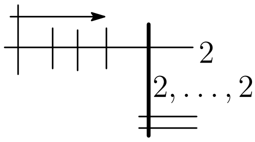

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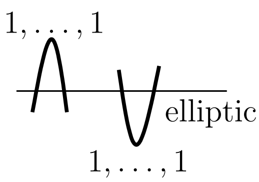

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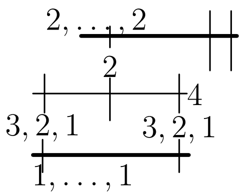

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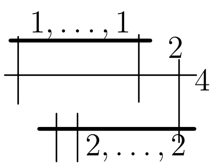

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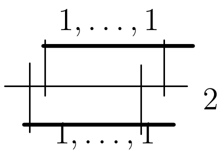

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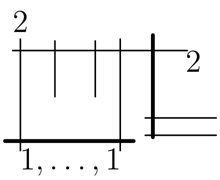

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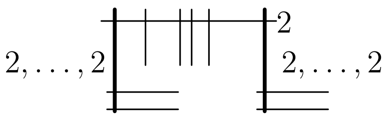
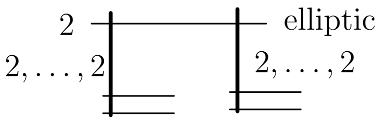


(viii7)



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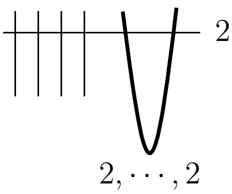

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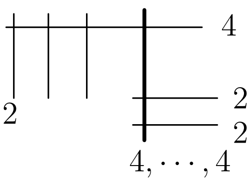

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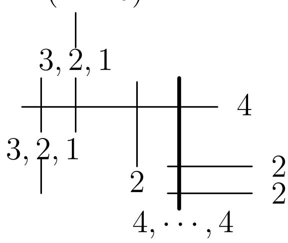

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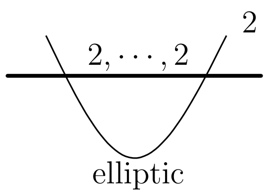

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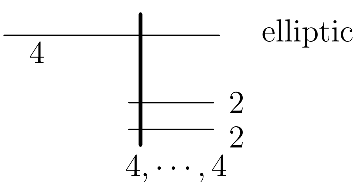

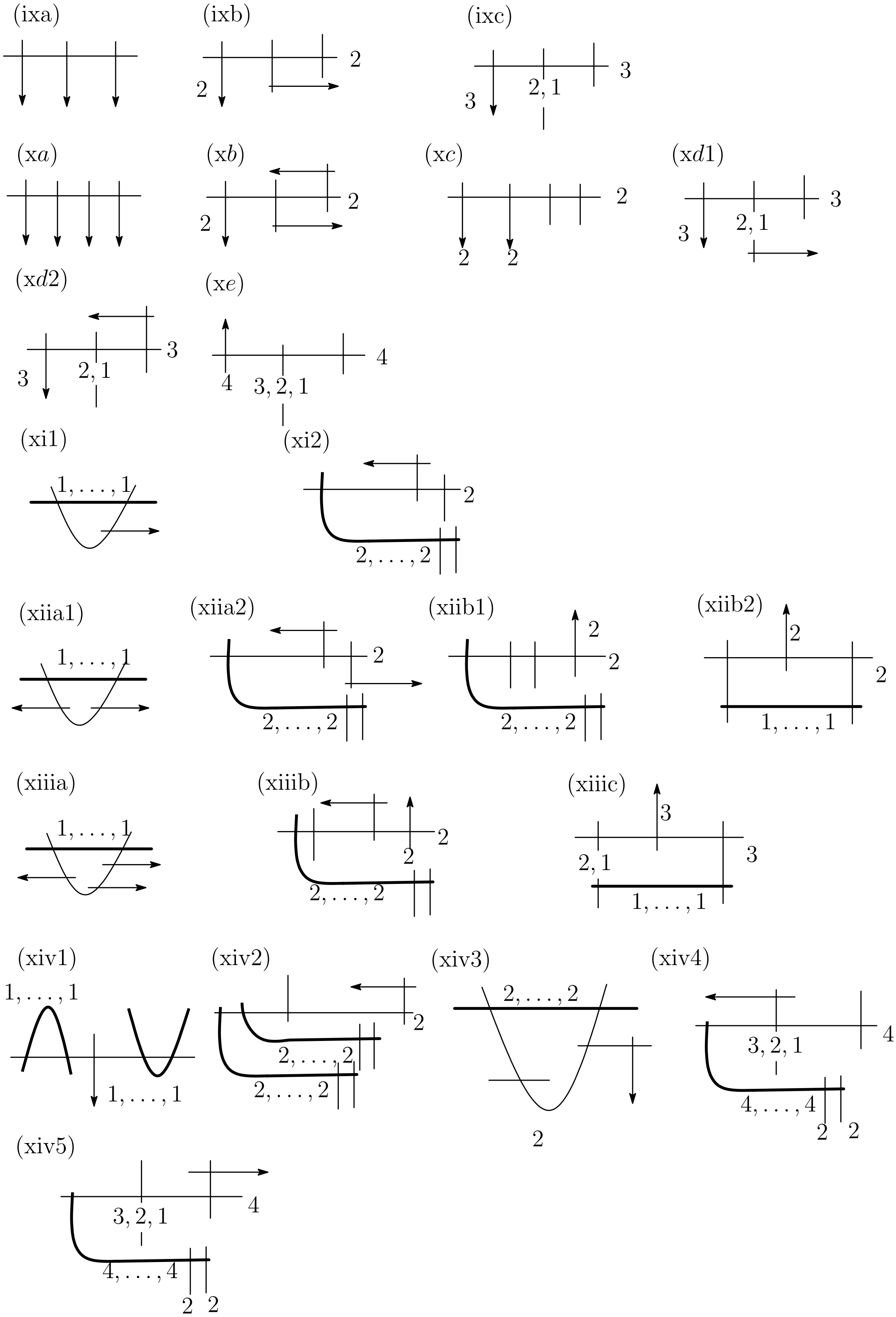
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(xd2)
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(xe)
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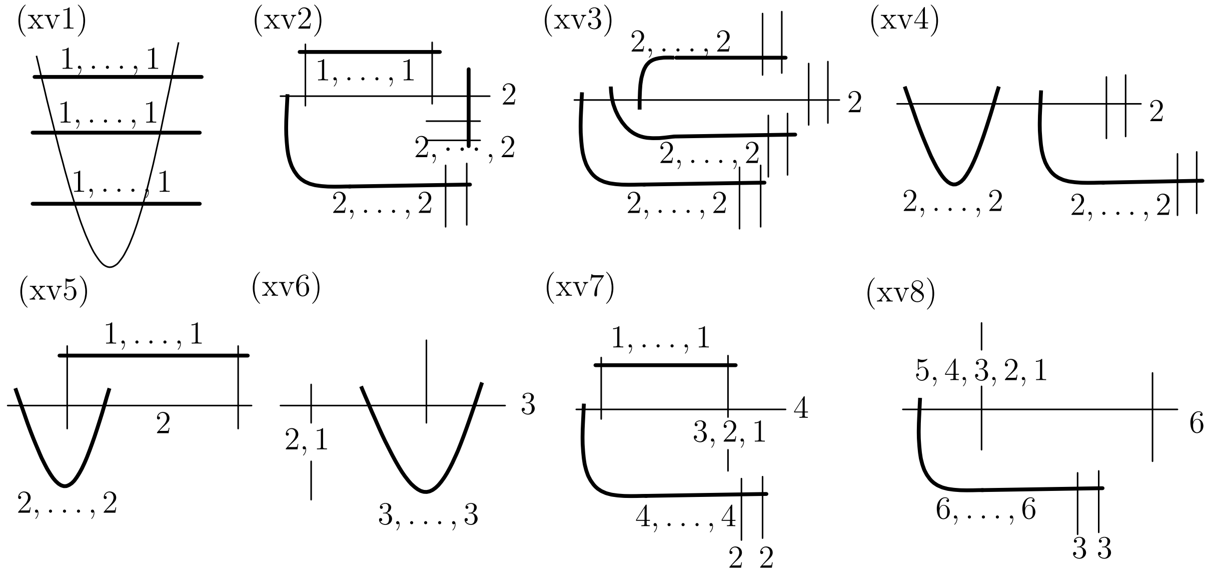
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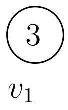
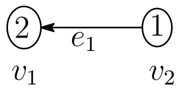
3 3

Table 2 Admissible system of cut curves

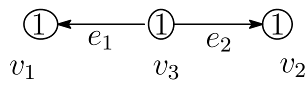
(A)



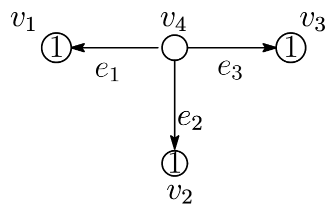
(B)



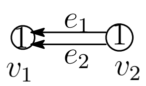
(C)



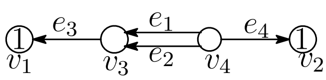
(D)



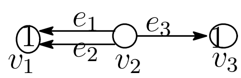
(E)



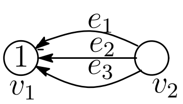
(F)



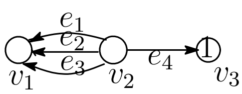
(G)



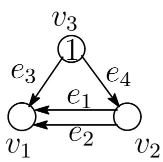
(H)



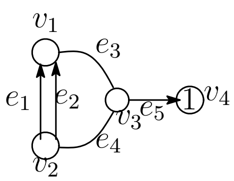
(I)



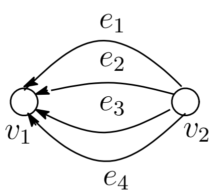
(J)



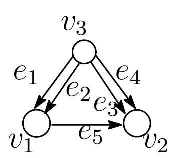
(K)



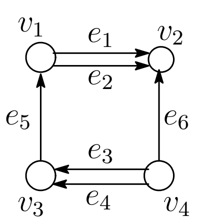
(L)



(M)



(N)



(O)

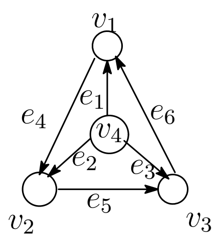
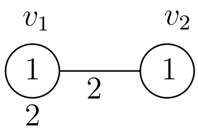
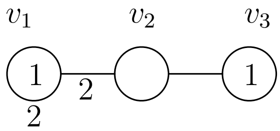


Table 3 Resolution of quotient graph

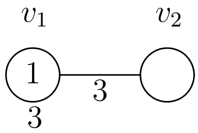
(1) $C_{iij}/II(1,1)$



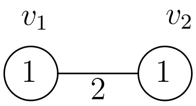
(2) $D_{iij}/II(1,1)$



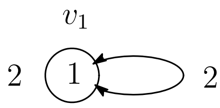
(3) $D_{iii}/III(1,1)$



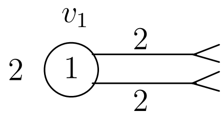
(4) $E_{ij}/II(0,1)$



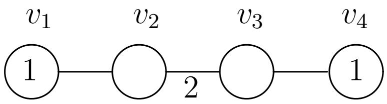
(5) $E_{ii}/II(1,2)$



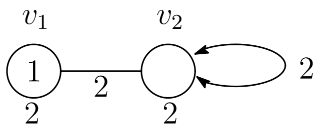
(6) $E_{ii}/II(1,3)$



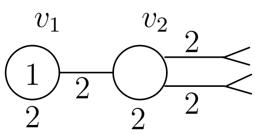
(7) $F_{ij}/II(0,1)$



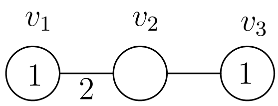
(8) $F_{ii}/II(2,1)$



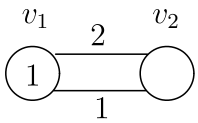
(9) $F_{ii}/II(2,2)$



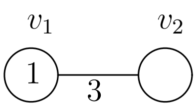
(10) $G_{ij}/II(0,1)$



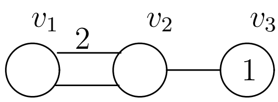
(11) $H_i/II(0,1)$



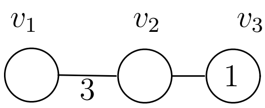
(12) $H_i/III(0,1)$



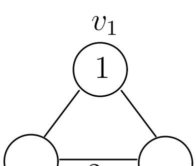
(13) $I_i/II(0,1)$



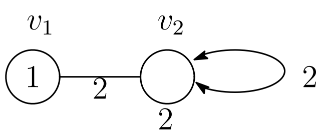
(14) $I_i/III(0,1)$



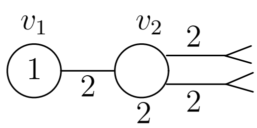
(15) $J_i/II(0,1)$



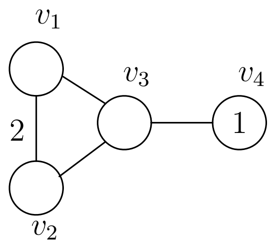
(16) $J_i/II(1,4)$



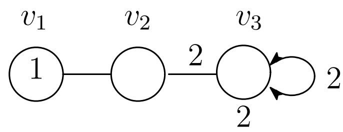
(17) $J_i/II(1,6)$



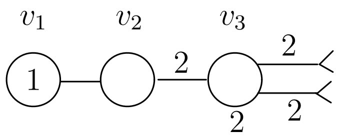
(18) $K_i/II(0,1)$



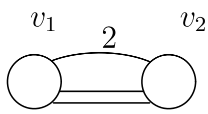
(19) $K_i/II(1,4)$



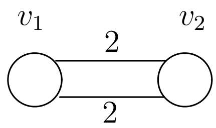
(20) $K_i/II(1,6)$



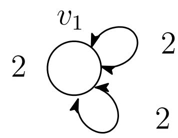
(21) $L/II(0,1)$



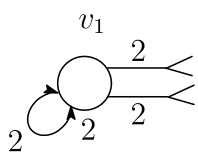
(22) $L/II(0,2)$



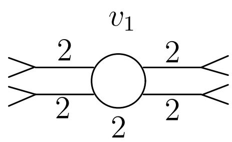
(23) $L/II(1,5)$



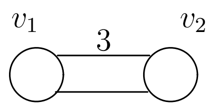
(24) $L/II(1,7)$



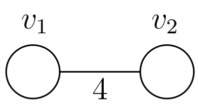
(25) $L/II(1,8)$



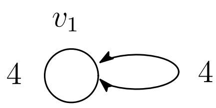
(26) $L/III(0,1)$



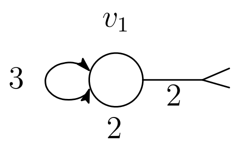
(27) $L/IV(0,1)$



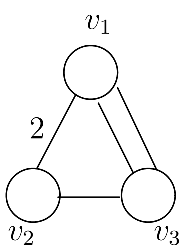
(28) $L/IV(1,1)$



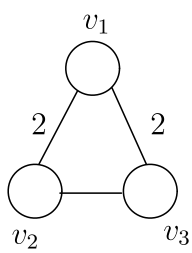
(29) $L/VI(1,1)$



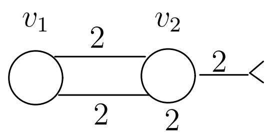
(30) $M/II(0,1)$



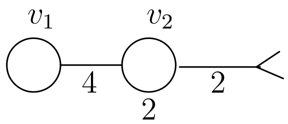
(31) $M/II(0,2)$



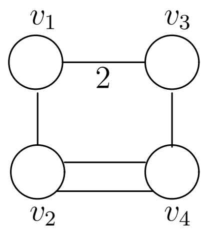
(32) $M/II(1,9)$



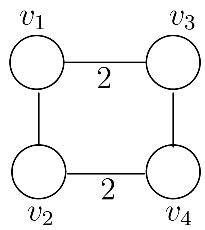
(33) $M/IV(1,2)$



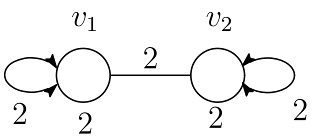
(34) $N/II(0,1)$



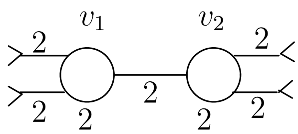
(35) $N/II(0,2)$



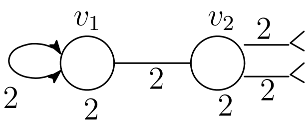
(36) $N/IV(0,1)$



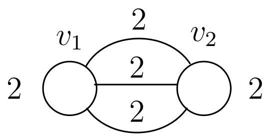
(37) $N/II(2,4)$



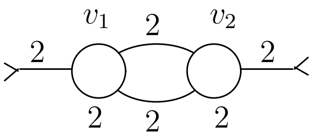
(38) $N/II(2,5)$



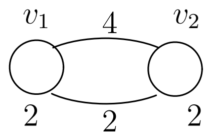
(39) $N/II(2,7)$



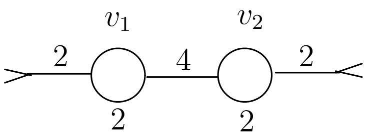
(40) $N/II(2,8)$



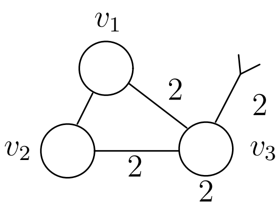
(41) $N/IV(2,1)$



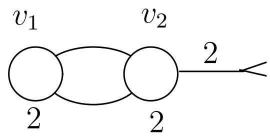
(42) $N/IV(2,2)$



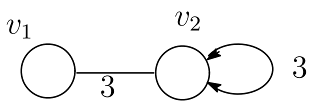
(43) $O/II(1,10)$



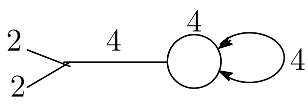
(44) $O/II(2,6)$



(45) $O/III(1,2)$



(46) $O/IV(1,3)$



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